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REDUCED THIN-LAYER ELEMENT MODEL FOR JOINT DAMPING

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Abstract. *Damping properties of assembled structures are largely influenced by frictional damping between joint interfaces. Therefore, these effects must be considered during the modelling process. Applying thin-layer elements (TLEs) with a linear, orthotropic material model on mechanical interfaces to incorporate joint damping has shown good agreement with experimental modal analysis in previous work. In the TLE model, constant hysteretic damping is assumed. The damping and stiffness parameters for the TLEs are experimentally identified on an isolated lap joint. Imprecisions caused by model simplifications and parameter uncertainty are addressed by model updating or uncertainty analysis. This requires multiple evaluations of systems that are equivalent besides their TLE parametrization. In this work, a model reduction technique for the thin-layer element modelling approach is presented which significantly reduces computational cost for the re-calculation of eigenvalues after joint parameters are changed. The reduction is based on an eigensensitivity analysis and results in a single, linear equation for each eigenvalue. The presented approach is applied to a model updating example. Here, the model reduction allows for a larger number of design variables. Therefore, experimental results can be reproduced more accurately.*

1 INTRODUCTION

While eigenfrequencies and mode shapes of assembled structures can be predicted reliably with the Finite Element Method, the incorporation of damping is subject to ongoing research. The key to valid damping predictions of assembled structures is accurate representation of joint damping caused by friction on mechanical interfaces. A proposed modelling technique for joint damping is the thin-layer element (TLE) approach which has shown promising results in previous work [1, 2]. Thereby, a thin layer of finite elements is placed on the joint interface. These TLEs contain a linear, phenomenological model of the joint behaviour. The major advantage of this approach is that the main contributions to energy dissipation in assembled structures are incorporated spatially correct while efficient evaluation of the model through eigenvalue analysis is still possible. Additionally, the linear approach facilitates comparison with data from experimental modal analysis. On the other hand, experimental investigations clearly show the non-linear behaviour of bolted joints [3]. Thus, the accuracy of the linear TLE approach is limited. The main source of inaccuracy is parameter and model uncertainty connected to the joint loss factor. To mitigate the effects of these inaccuracies, model updating [4] or uncertainty analysis [5] are employed. That, however, requires a large number of model evaluations with varying loss factors.

In this work, a model reduction technique for the TLE modelling approach and an application to model updating are presented. The reduction is based on an eigensensitivity analysis and takes advantage of the particularly simple formulation of joint damping in the form of constant hysteretic damping. The result is a single, linear equation for each eigenvalue.

2 THIN-LAYER ELEMENT JOINT MODEL

This section covers the basic approach of joint modelling using TLEs. Further details can be found in [2, 6]. As mentioned above, the basic idea of the TLE modelling approach is to place a layer of elements with a linear, orthotropic material model on all joint interfaces (Fig. 1). The

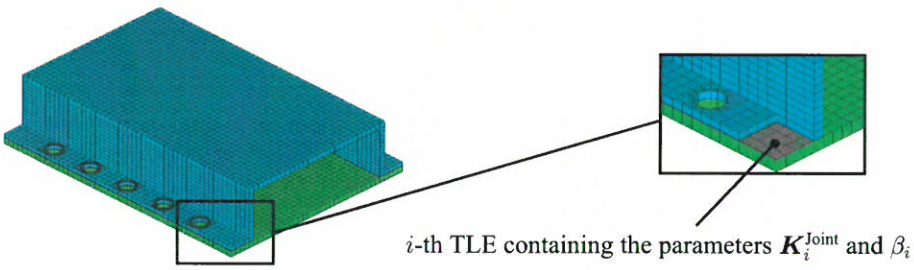

Figure 1: FE model of the test structure with exposed thin-layer elements.

TLEs reproduce the localized occurrence of joint damping in assembled structures in contrast to e.g. Rayleigh Damping where the energy dissipation is distributed over the entire structure. The TLEs are implemented as hexahedral elements with a height to width ratio of up to 1:100. As joints have distinct behaviour in normal and tangential direction with respect to the interface, an orthotropic material is model employed. Stiffness and damping parameters of the joint are experimentally identified on a specialized experimental setup [6].

Joint damping shows only weak dependence on frequency. Therefore, the model of constant hysteretic damping is applied [7]. The starting point for the implementation of constant hysteretic damping is the discrete equation of motion of a free, undamped system

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{0}\,. \tag{1}$$

Here, $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{K}$ the real-valued stiffness matrix and $\boldsymbol{u}$ the displacement vector. Constant hysteretic damping can now be incorporated by replacing the real-valued stiffness matrix $\boldsymbol{K}$ with a complex-valued stiffness matrix $\boldsymbol{K}^*$ with

$$\boldsymbol{K}^* = \boldsymbol{K} + \mathrm{j}\sum_{i=1}^{n} \alpha_i \boldsymbol{K}_i^{(\text{Material})} + \mathrm{j}\sum_{i=1}^{m} \beta_i \boldsymbol{K}_i^{(\text{Joint})}\,, \tag{2}$$

where $\boldsymbol{K}_i^{(\text{Material})}$ are the regular element stiffness matrices, $\boldsymbol{K}^{(\text{Joint})}$ are the TLE stiffness matrices and j is the complex unit. The material loss factor is represented by α_i and the joint loss factor by β_i. While material and joint damping are accounted for by this approach, joint damping is decisive for accurate damping predictions. Thus, further elaboration is limited to the joint loss factor β_i.

Replacing $\boldsymbol{K}$ in Eq. (1) by $\boldsymbol{K}^*$ from Eq. (2), the eigenvalue problem can be formulated

$$(\lambda_k^2 \boldsymbol{M} + \boldsymbol{K}^*)\boldsymbol{\psi}_k = \boldsymbol{0}, \tag{3}$$

with eigenvalues λ_k and eigenvectors $\boldsymbol{\psi}_k$.

3 THIN-LAYER ELEMENT MODEL REDUCTION

3.1 Derivation

As mentioned above, models with TLEs often require multiple evaluations of Eq. (3) for systems that are equivalent in all respects but the joint loss factors β_i. In this chapter, a more efficient way to calculate the resulting eigenvalues in the sense of a model reduction is presented. For the sake of brevity, all derivations will be shown for λ_k^2. Modal damping is presented in terms of $\mathrm{Im}(\lambda_k^2)$. Expansion to λ_k is possible at each step.

First, two systems are introduced. The base system with the system matrices $\boldsymbol{M}$ and $\boldsymbol{K}$, and an augmented system represented by $\hat{\boldsymbol{M}}$ and $\hat{\boldsymbol{K}}$ (henceforth $\hat{\bullet}$ denotes affiliation to the augmented system). The two systems differ only in joint damping which means

$$\hat{\boldsymbol{M}} = \boldsymbol{M} \quad \text{and} \tag{4}$$

$$\hat{\boldsymbol{K}}^* = \boldsymbol{K}^* + \mathrm{j}\sum_{i=1}^{m} \Delta\beta_i \boldsymbol{K}_i^{(\text{Joint})}\,. \tag{5}$$

$\Delta\beta_i$ denotes the difference of the loss factor between the augmented system and the base system

$$\Delta\beta_i = \hat{\beta}_i - \beta_i\,. \tag{6}$$

Eigensensitivity analysis is now employed to find a relation between the change of the i-th loss factor $\Delta\beta_i$ and the k-th squared eigenvalue $\hat{\lambda}_k^2$ of the augmented system.

Rearranging Eq. (3) into its Rayleigh Quotient [8] form and scaling $\boldsymbol{\psi}_k$ such that

$$\boldsymbol{\psi}_k^T \boldsymbol{M} \boldsymbol{\psi}_k = 1\,, \tag{7}$$

one obtains

$$\lambda_k^2 = -\boldsymbol{\psi}_k^T \boldsymbol{K}^* \boldsymbol{\psi}_k\,. \tag{8}$$

Using the definition of the complex-valued stiffness matrix $\boldsymbol{K}^*$ in Eq. (2), the partial derivative of Eq. (8) with respect to β_i yields

$$\frac{\partial \lambda_k^2}{\partial \beta_i} = -\boldsymbol{\psi}_k^T(\mathrm{j}\boldsymbol{K}_i^{(\mathrm{Joint})})\boldsymbol{\psi}_k = J_{ik}\,. \tag{9}$$

A more detailed derivation of this result can be found in [9, 10, 11]. Equation (9) reveals that the derivative of λ_k^2 with respect to a joint loss factor β_i only depends on the corresponding (constant) stiffness matrix $\boldsymbol{K}_i^{\mathrm{Joint}}$ and eigenvector $\boldsymbol{\psi}_k$. In typical applications, the TLEs have negligible influence on the mode shape since they constitute only a small fraction of the overall structure. Hence it is feasible to assume

$$\frac{\partial \boldsymbol{\psi}_k}{\partial \beta_i} \approx \boldsymbol{0}\,. \tag{10}$$

Therefore, J_{ik} is independent of β_i and Eq. (9) can be used to formulate a reduced model:

$$\hat{\lambda}_k^2 = \lambda_k^2 + \sum_{i=1}^{m} J_{ik}\Delta\beta_i\,. \tag{11}$$

With Eq. (11), eigenvalues of the augmented system can be calculated from a single, linear equation. One evaluation of the full base model is necessary to calculate λ_k^2 and $\boldsymbol{\psi}_{\boldsymbol{k}}$. During this evaluation, all TLE stiffness matrices $\boldsymbol{K}_i^{\mathrm{Joint}}$ are assembled as well and can be stored for the calculation of $\boldsymbol{J}_{ik}$.

As long as Eq. (10) is a sufficiently exact approximation, Eq. (11) is valid for all reasonable values of $\Delta\beta_i$. One can further assume that the eigenvectors, though in reality complex, are approximately real-valued. Therefore, J_{ik} is purely imaginary and Eq. (11) reveals that a change of the loss factor β_i only affects the imaginary part of $\hat{\lambda}_k^2$. This is equivalent with the intuitive assumption, that a variation of the loss factor mainly affects modal damping ratios without significantly altering eigenfrequencies.

3.2 Validation

The reduced model is validated by examining the first eight eigenvalues of the test structure depicted in Fig. 1. To that end, the eigenvalues of the system are calculated for different loss factors by evaluating the full model (Eq. (3)) and by using the reduced model (Eq. (11)). It is important to note that here, in contrast to the model updating example in Section 4, all TLEs have identical loss factors β_i. Figure 2 shows the modal damping (in terms of $\mathrm{Im}(\lambda_k^2)$) for the first eight modes for loss factors ranging between 0 and 1. The solid line depicts the simulation with the reduced model and the dots represent values from full model evaluations. The base system used for the model reduction has a loss factor of $\beta_i = 0.05$ which is close to the lower limit of the parameter range. Nonetheless, all deviations from values calculated with the full model are smaller than $0.15\ \%$.

4 MODEL UPDATING WITH REDUCED MODEL

The TLE modelling approach was introduced as an efficient way to predict damping in assembled structures before a physical prototype exists. The original approach [2] employs an uniform loss factor parametrization across the joint surface. While this approach is effective for damping predictions, it is not well suited for model updating because only a single design parameter exists.

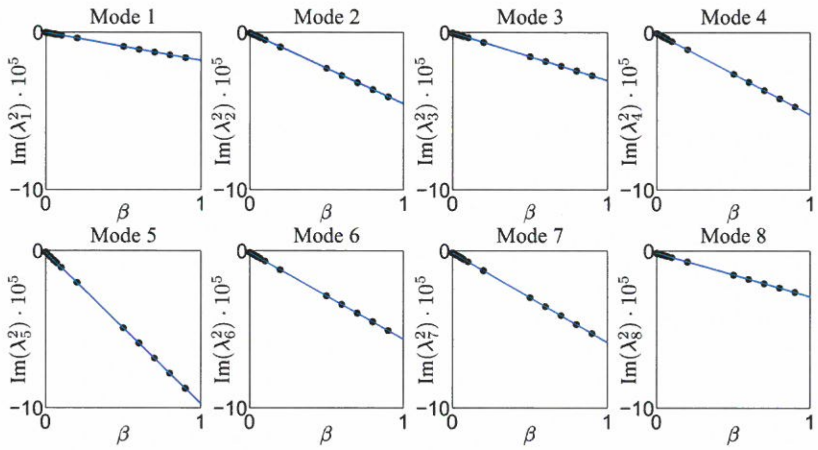

Figure 2: Validation of the reduced model by comparison of $\mathrm{Im}(\lambda_k^2)$ calculated by full model evaluations (dots) and reduced model evaluations (solid line).

With the reduced model, a more numerically compliant TLE approach can be devised. Instead of a single parameter for the entire joint interface, each element can be allocated an independent loss factor parameter. This increases the number of design variables for the optimization from one in the original approach, to several hundred (880 in the presented example). An optimization with such a high number of variables requires a large number of iterations which is only feasible under the utilization of the reduced model.

In the presented example, parameters for 880 TLEs are to be found such that the experimentally determined modal damping ratios for the first eight modes are reproduced in an optimal way. Due to the large number of design variables, this problem has various solutions with most of them being physically not feasible. Therefore, a set of boundary conditions has to be defined which reduces the set of solutions to physically reasonable ones.

As first boundary condition, the maximum difference of the loss factor of neighbouring elements $\Delta\beta_{\mathrm{max}}$ is limited. The loss factor depends on normal and tangential loads [12]. As these loads change gradually, the loss factor must follow accordingly. This is implemented as an inequality boundary condition. For two elements, i and j, which have at least one common node, the following inequalities are enforced:

$$|\beta_i - \beta_j| < \Delta\beta_{\mathrm{max}} \tag{12}$$

With the second boundary condition, symmetry is enforced. As stated before, the loss factor is determined by tangential and normal loads. Therefore, a geometrically symmetric assembly under symmetric loads must also have a symmetric joint parameter distribution. The example structure has two planes of symmetry. Figure 3 shows four elements, k, l, m, n which are assigned identical parameters.

As a final condition, the mean loss factor β_{mean} is kept constant. This condition is implemented as an equality constraint

$$\sum_i^n \frac{\beta_i}{n} = \beta_{\mathrm{mean}} \, . \tag{13}$$

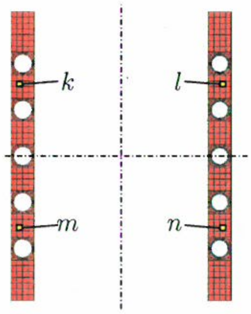

Figure 3: Joint interface of the example structure with four elements k, l, m, n that must be allocated identical loss factors.

It ensures that the optimization is not achieved by uniformly adapting the loss factor. Comparability of different models is maintained this way, too.

With these boundary conditions, the loss factor parameters are optimized such that the first eight experimentally determined modal damping ratios of the test structure are reproduced optimally in a least-squares sense. The algorithm converges after 1330 iterations in 58 seconds. At approximately 20 seconds per full model evaluation and numerous model evaluations per iteration, the cost advantage of the reduced model is significant. As base line, model updating with uniform parametrization is performed. This optimization with a single degree of freedom is representative of the performance of the original TLE modelling approach. In Figure 5, the relative error

$$\delta\,\mathrm{Im}(\lambda_k^2) = \left| \frac{\mathrm{Im}(\lambda_k^2) - \mathrm{Im}(\lambda_{k,\mathrm{exp}}^2)}{\mathrm{Im}(\lambda_{k,\mathrm{exp}}^2)} \right| \tag{14}$$

of the resulting modal damping values with respect to experimentally determined reference values $\mathrm{Im}(\lambda_{k,\mathrm{exp}}^2)$ is depicted. While it is evident that the original approach already yields good results, especially for lower order modes, the new approach improves the performance. Higher order modes have geometrically more complex mode shapes. The original, uniform parametrization approach cannot depict these conditions as accurately as the element-wise independent parametrization employed in the new approach. Comparing the loss factor parameter

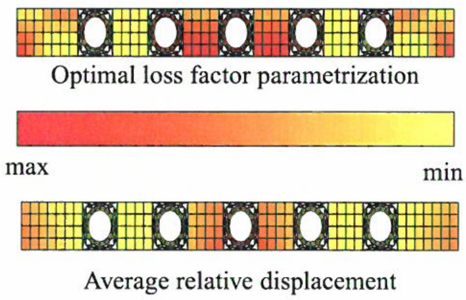

Figure 4: Comparison of the loss factor parametrization after model updating and the average relative displacement for the first eight modes.

distribution resulting from the new model updating approach with the average relative displacement over all relevant modes, a strong similarity is evident (Fig. 4). High relative displacement in the joint causes high energy dissipation [12]. Thus, Fig. 4 indicates that the optimization under the derived boundary conditions arrives at a physically meaningful solution.

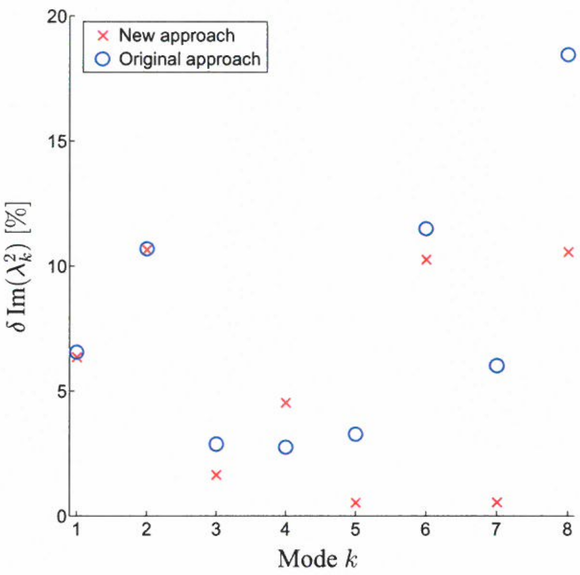

Figure 5: Relative error of the modal damping after model updating with the new and original approach.

5 CONCLUSIONS

The existing thin-layer element modelling approach is extended by a model reduction. The reduced model is derived through an eigensensitivity analysis and provides a single, linear equation for each eigenvalue. A prerequisite for the applicability of the presented technique is a negligible influence of the TLE parametrization on the eigenvectors.

Through the model reduction, the TLE modelling approach can be augmented for better performance in model updating applications. This approach reproduces experimental data more accurately by providing a very large number of design variables to the optimization algorithm. Several boundary conditions are defined in order to generate physically meaningful results. While the original TLE approach localizes joint damping at mechanical interfaces but does not consider variations over the area of the interface, the new model updating approach identifies areas of high and low energy dissipation on the interface.

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