University of Stuttgart
Germany

Institute for
Nonlinear Mechanics

Why Rockshape matters

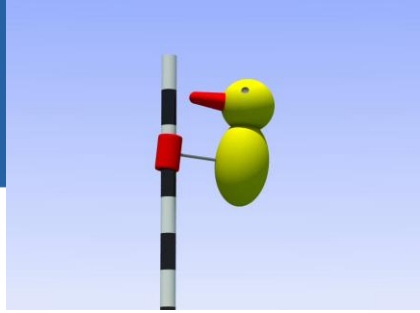
Full 3D Rockfall Simulation using Nonsmooth Dynamics

Remco Leine



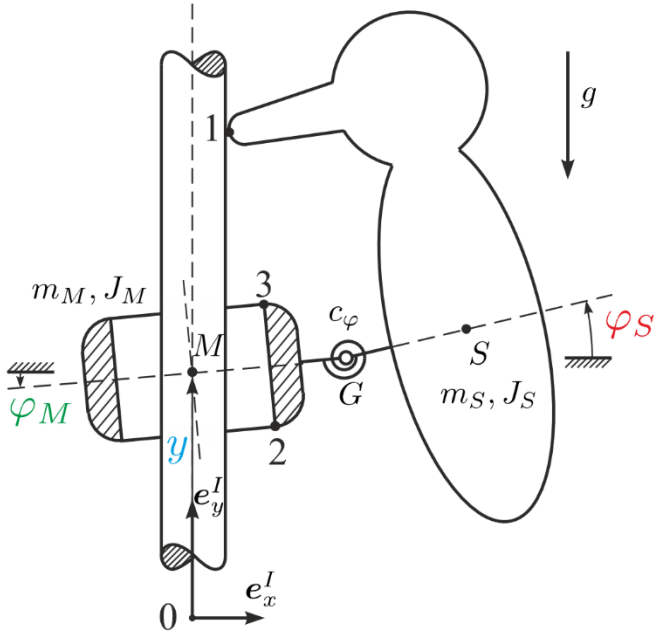
Gurtnellen (CH),
2006

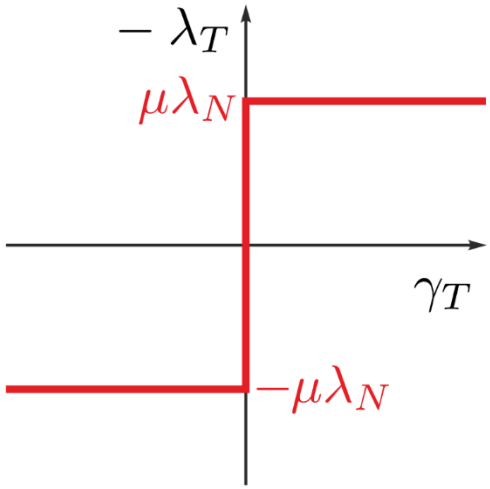
Nonsmooth Dynamics

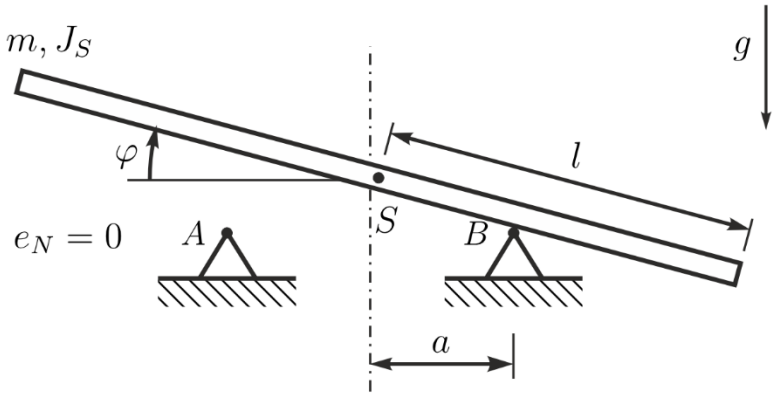


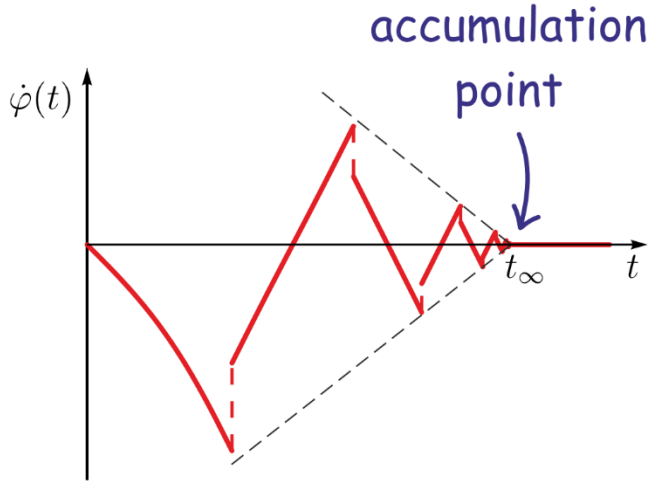
Formalism for multibody dynamics with frictional unilateral constraints based on:

- convex analysis
- set-valued force & impact laws
- numerical simulation methods:
  - event-driven integration method
  - Moreau-type timestepping methods

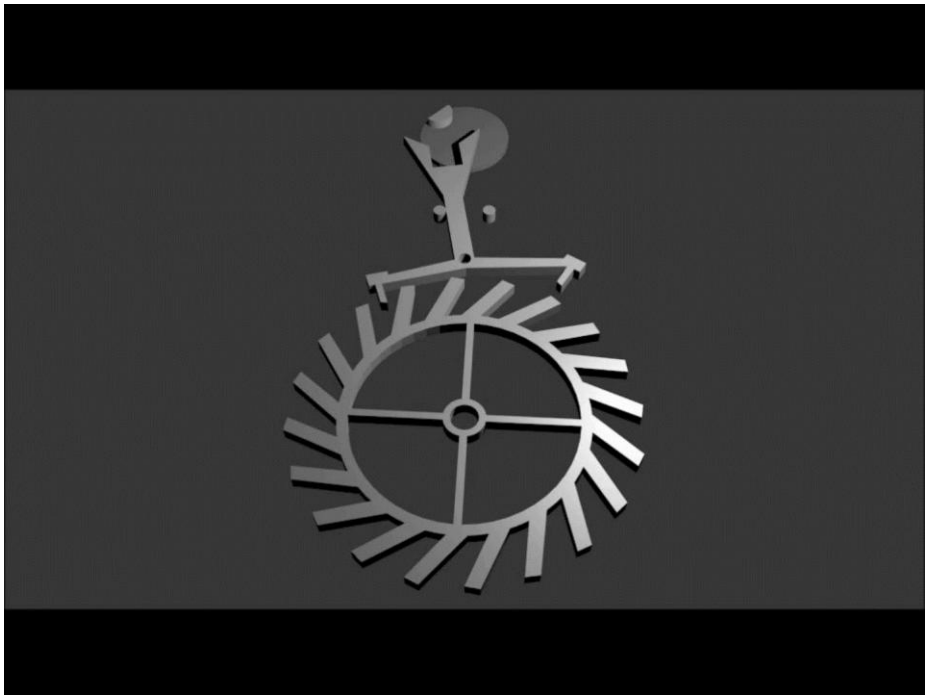


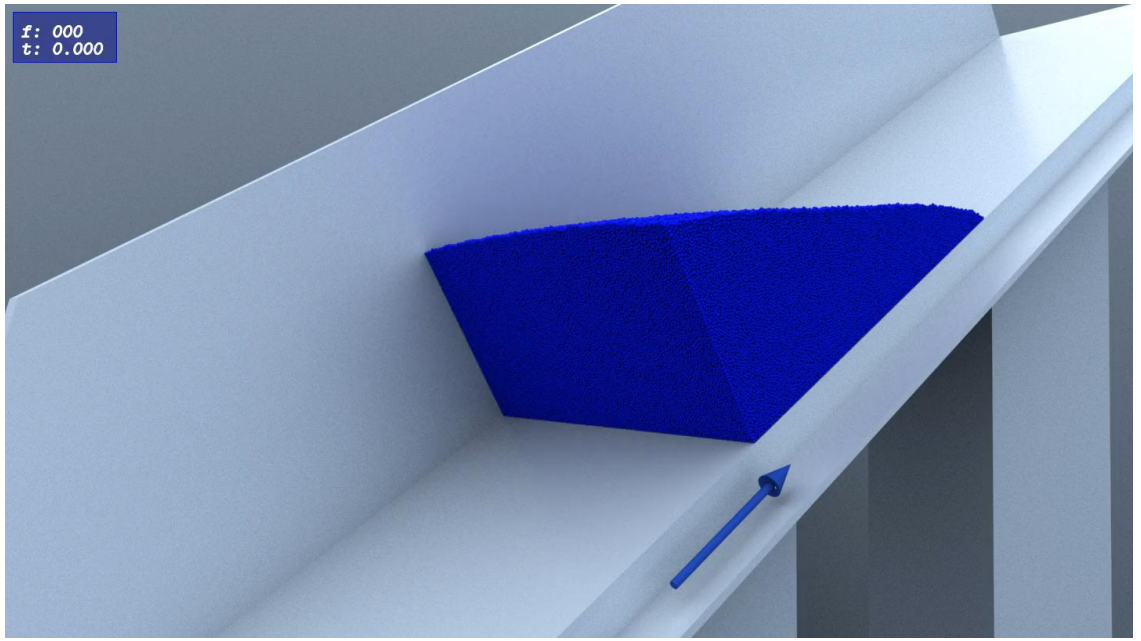






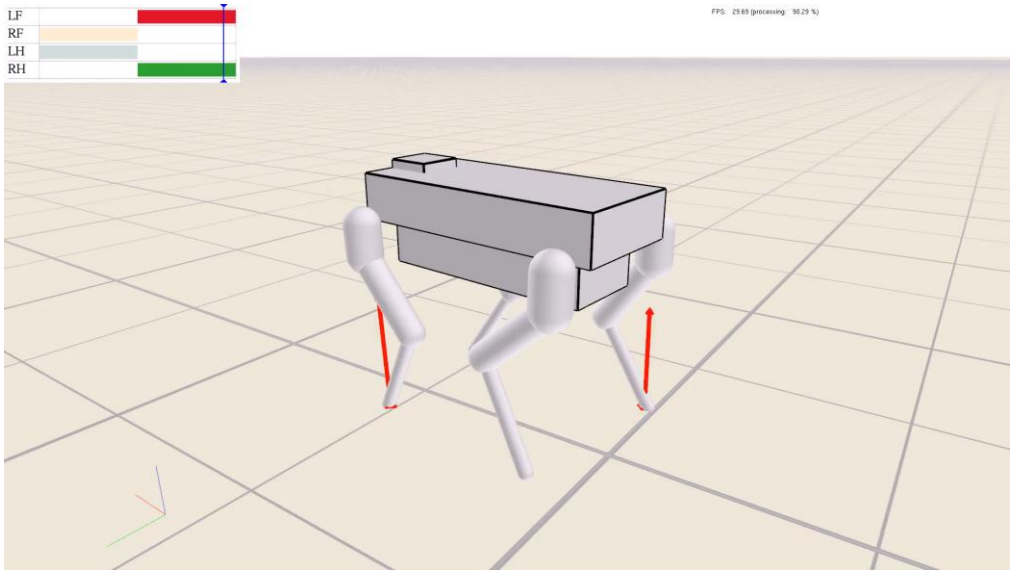
Simulations using Nonsmooth Dynamics







Robotics
Granular media
Vibration analysis
Contact mechanics
…



Rockfall



The rockfall hazard



Ronchi di Termeno (IT), 2014

Corominas et al. (2025), *Geoenvironmental Disasters*

1,118 non-seismic reported rockfall events (1803 - 2021) in Spain causing

- 1,550 deaths
- 2,184 injuries

+ damage to infrastructure

worldwide?

Rockfall protection measures



galleries

dams



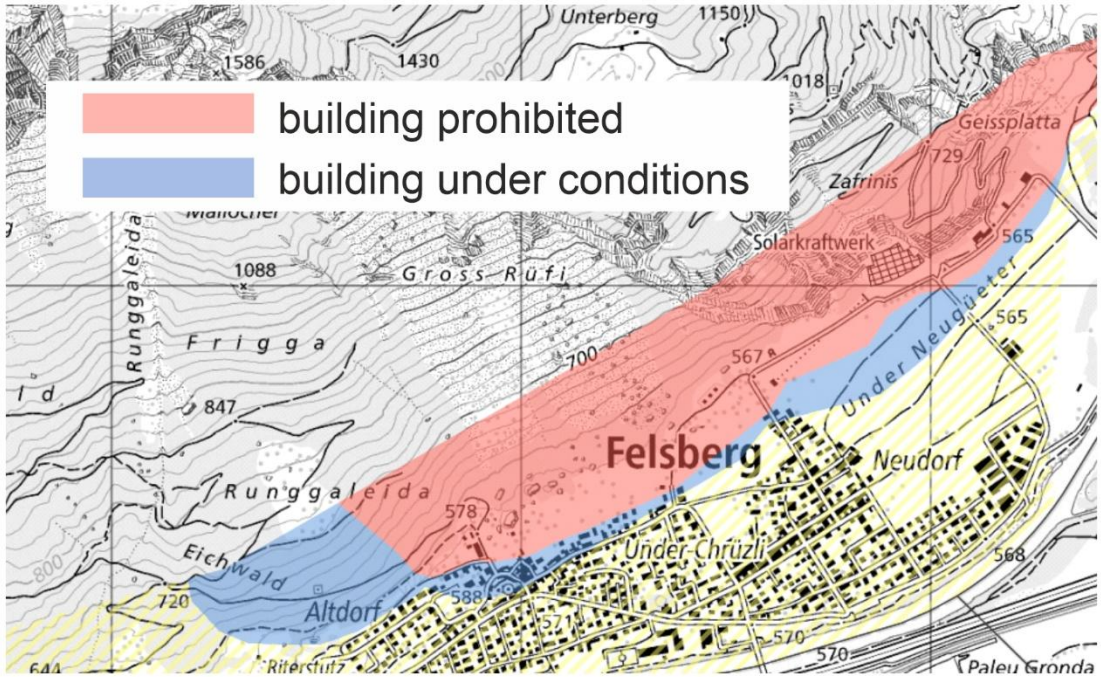


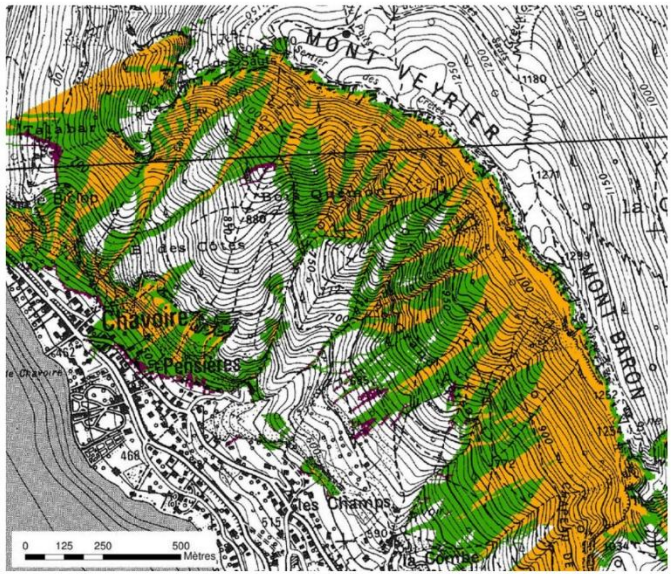
nets

forest

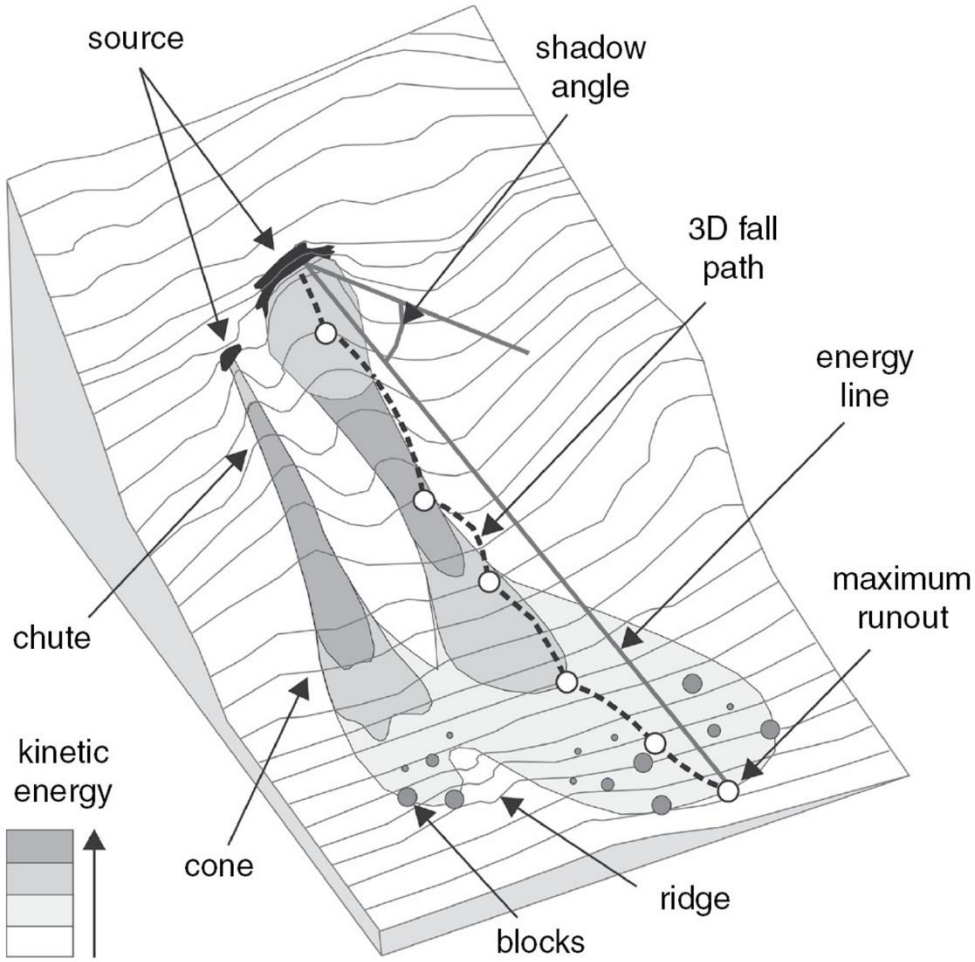


Rockfall hazard assessment

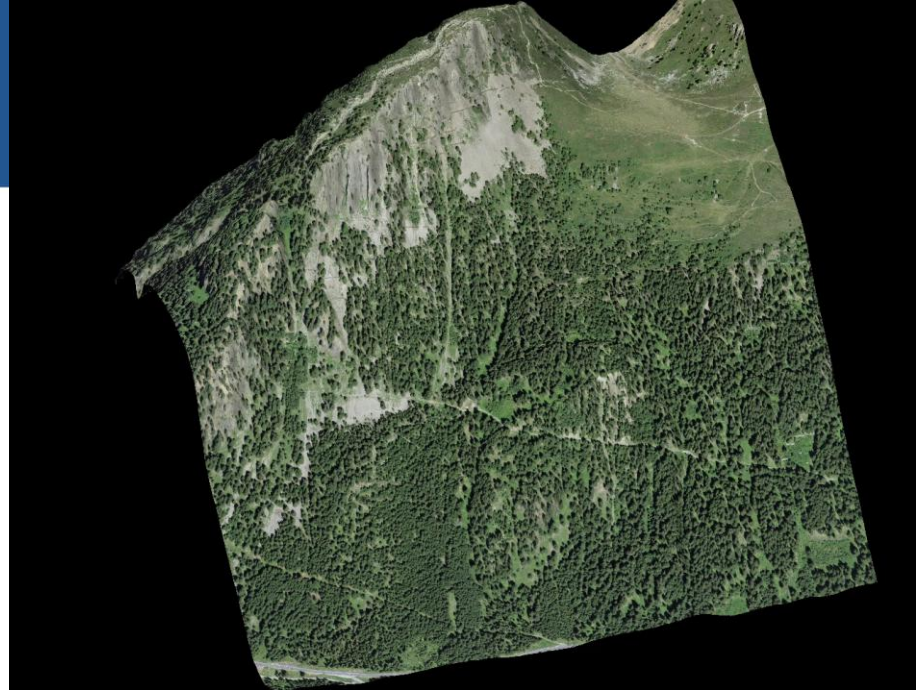




Monnet et al., 2010



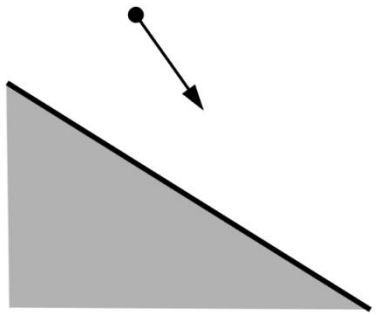
Trajectory simulation



Monte-Carlo simulations varying:

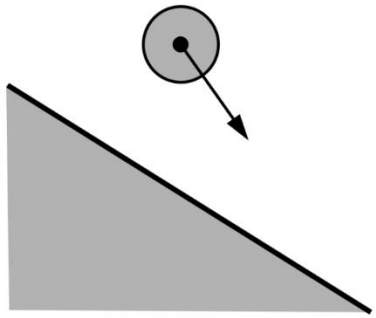
- rock mass
- rock shape
- rock initial position in source zone
- initial orientation

point mass model



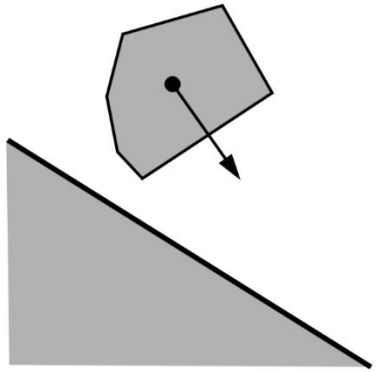
- no rock geometry
- no rotational velocity
- no distinction between rotational and translational kinetic energy

sphere model



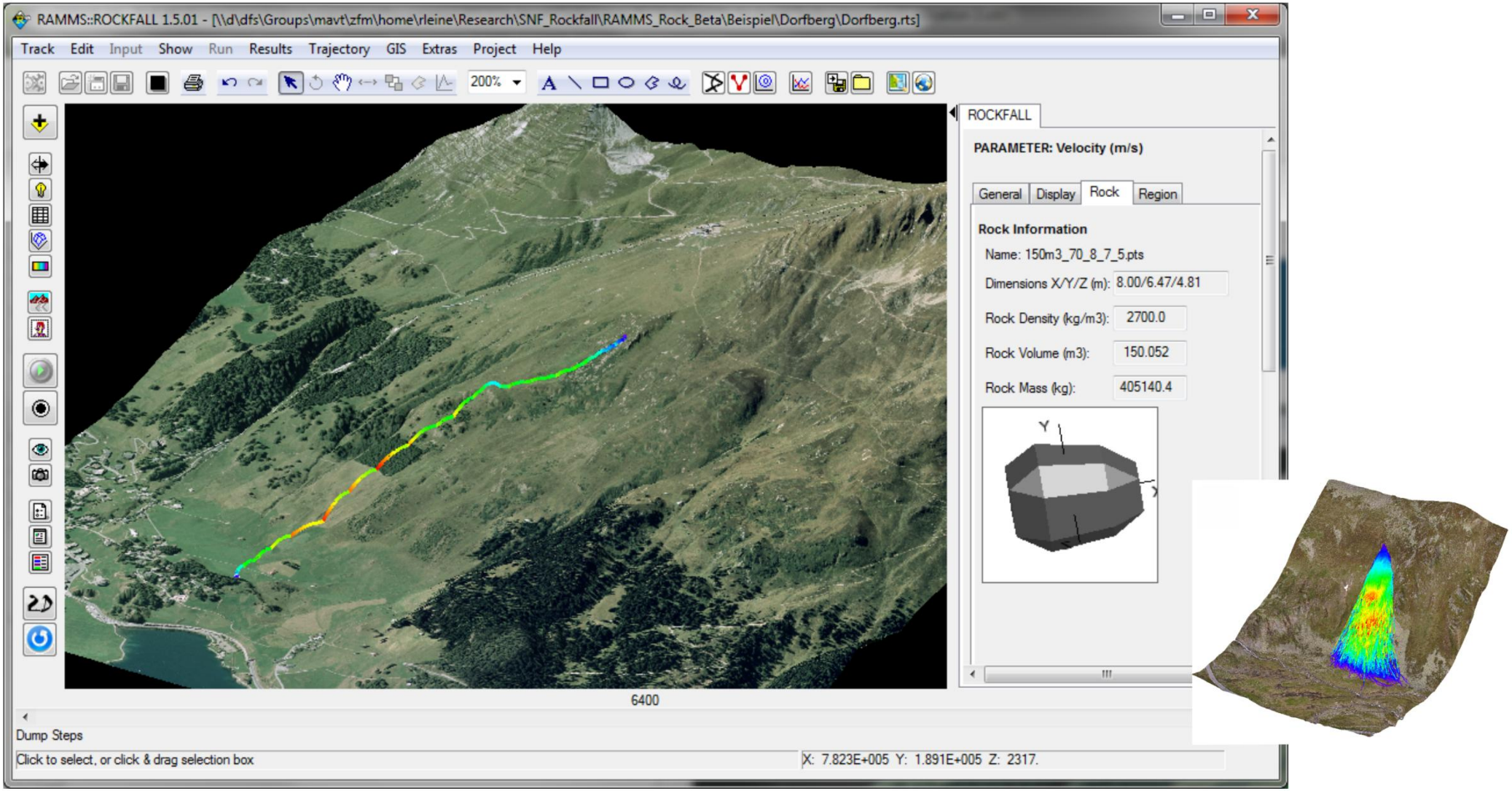
- only one rock geometry
- rock rolls too far
- no gyroscopic effects

complex shape model



- arbitrary rock geometry
- computationally expensive
- shape effects
- gyroscopic effects

Rockfall hazard mapping using RAMMS::ROCKFALL

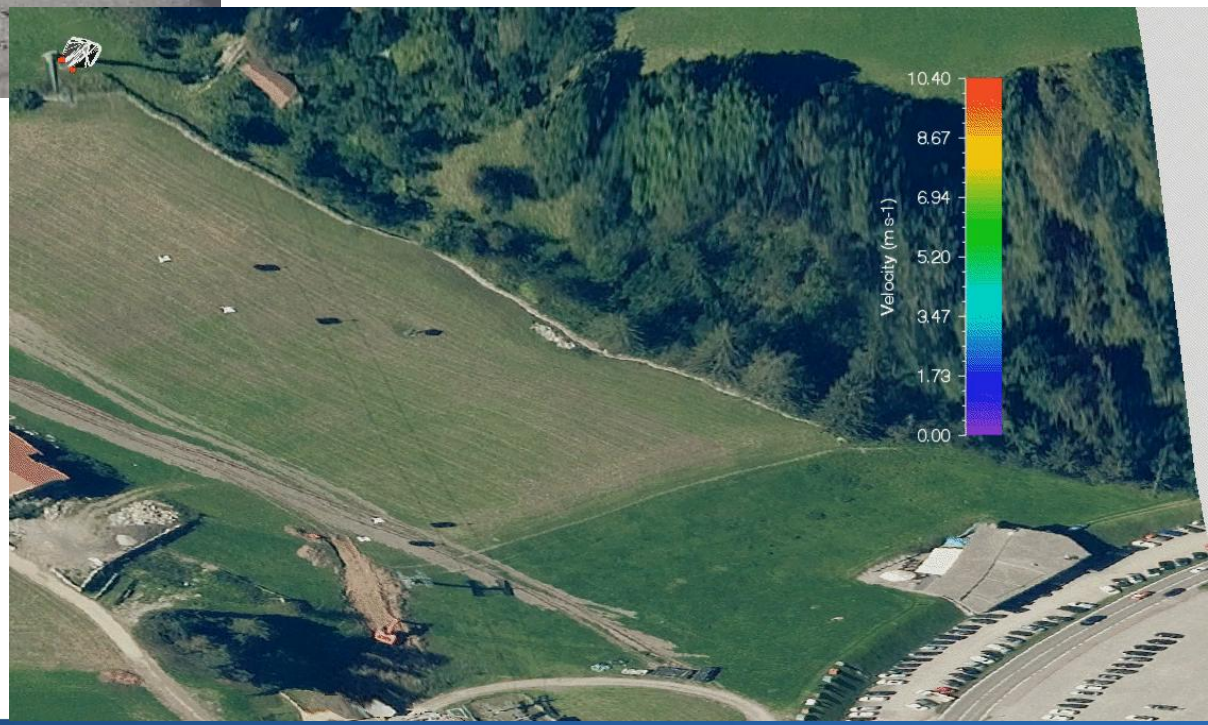


Leine, Schweizer, Christen, Glover, Bartelt, and Gerber: "Simulation of rockfall trajectories with consideration of rock shape", Multibody System Dynamics, 32, pp. 241–271, 2014.

The importance of rock shape: sliding vs. rolling



security camera
base station ski lift Speikboden
Sand in Taufers, South Tyrol
2023



RAMMS::Rockfall simulation
by **Joël Borner**

Rock model

Rock shape

point cloud → Convex hull algorithm

- convex hull
- hyperplane equations

vertex-representation

$\mathrm{conv}(P_1, P_2, \ldots)$

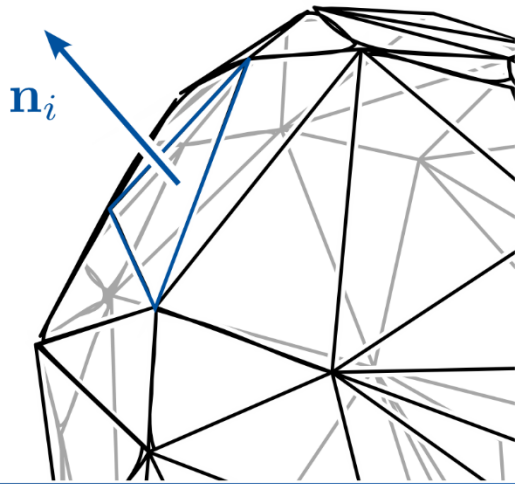
halfspace-representation

$C = \cap_i \left\{ \mathbf{x} \in \mathbb{R}^3 | \mathbf{n}_i^{\mathrm{T}} \mathbf{x} + d_i \leq 0 \right\}$



Further calculations

- outward pointing unit normals
- volume, center of mass
- inertia
- principal axes frame



Rock model

Rock shape

point cloud → Convex hull algorithm

- convex hull
- hyperplane equations

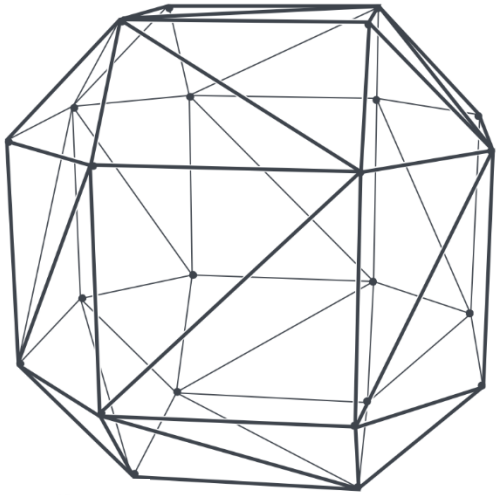
vertex-representation

$\mathrm{conv}(P_1, P_2, \ldots)$

halfspace-representation

$C = \cap_i \left\{ \mathbf{x} \in \mathbb{R}^3 | \mathbf{n}_i^{\mathrm{T}} \mathbf{x} + d_i \leq 0 \right\}$

EOTA rock shape

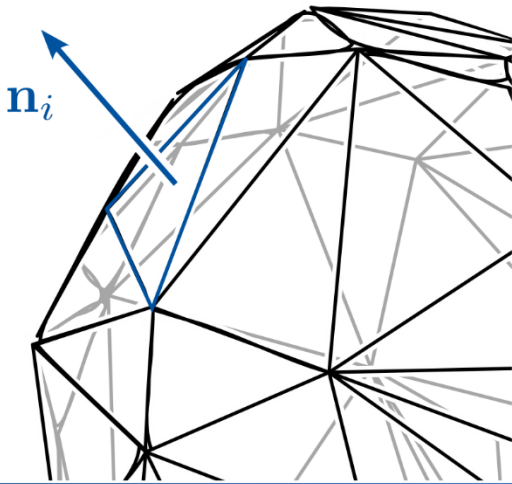


Further calculations

- outward pointing unit normals
- volume, center of mass
- inertia
- principal axes frame



realistic rock shape



Rock model: equations of motion

quaternion: $\mathbf{p} = \begin{pmatrix} e_0 & \mathbf{e}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}} \in \mathbb{R}^4, \quad \mathbf{e} = \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix}^{\mathrm{T}}, \quad \|\mathbf{p}\| = 1$

rotation matrix: $\mathbf{R} = (2e_0^2 - 1)\mathbf{I} + 2(\mathbf{e}\mathbf{e}^{\mathrm{T}} + e_0\tilde{\mathbf{e}}), \quad {}_I\vec{\mathbf{x}} = \mathbf{R}\,{}_K\vec{\mathbf{x}}$

$\dot{\mathbf{p}} = \mathbf{H}(\mathbf{p})\,{}_K\vec{\boldsymbol{\Omega}}, \qquad \mathbf{H}(\mathbf{p}) = \frac{1}{2}\begin{pmatrix} -\mathbf{e} & -\tilde{\mathbf{e}} + e_0\mathbf{I} \end{pmatrix}^{\mathrm{T}}$

generalised coord./veloc.: $\mathbf{q} = \begin{pmatrix} {}_I\vec{\mathbf{r}}_{OS}^{\mathrm{T}} & \mathbf{p}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}} \in \mathbb{R}^7 \qquad \mathbf{u} = \begin{pmatrix} {}_I\dot{\vec{\mathbf{r}}}_{OS}^{\mathrm{T}} & {}_K\vec{\boldsymbol{\Omega}}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}} \in \mathbb{R}^6$

linear momentum eq. : $m\,{}_I\ddot{\vec{\mathbf{r}}}_{OS} = {}_I\vec{\mathbf{F}}^a$ inertial frame: I

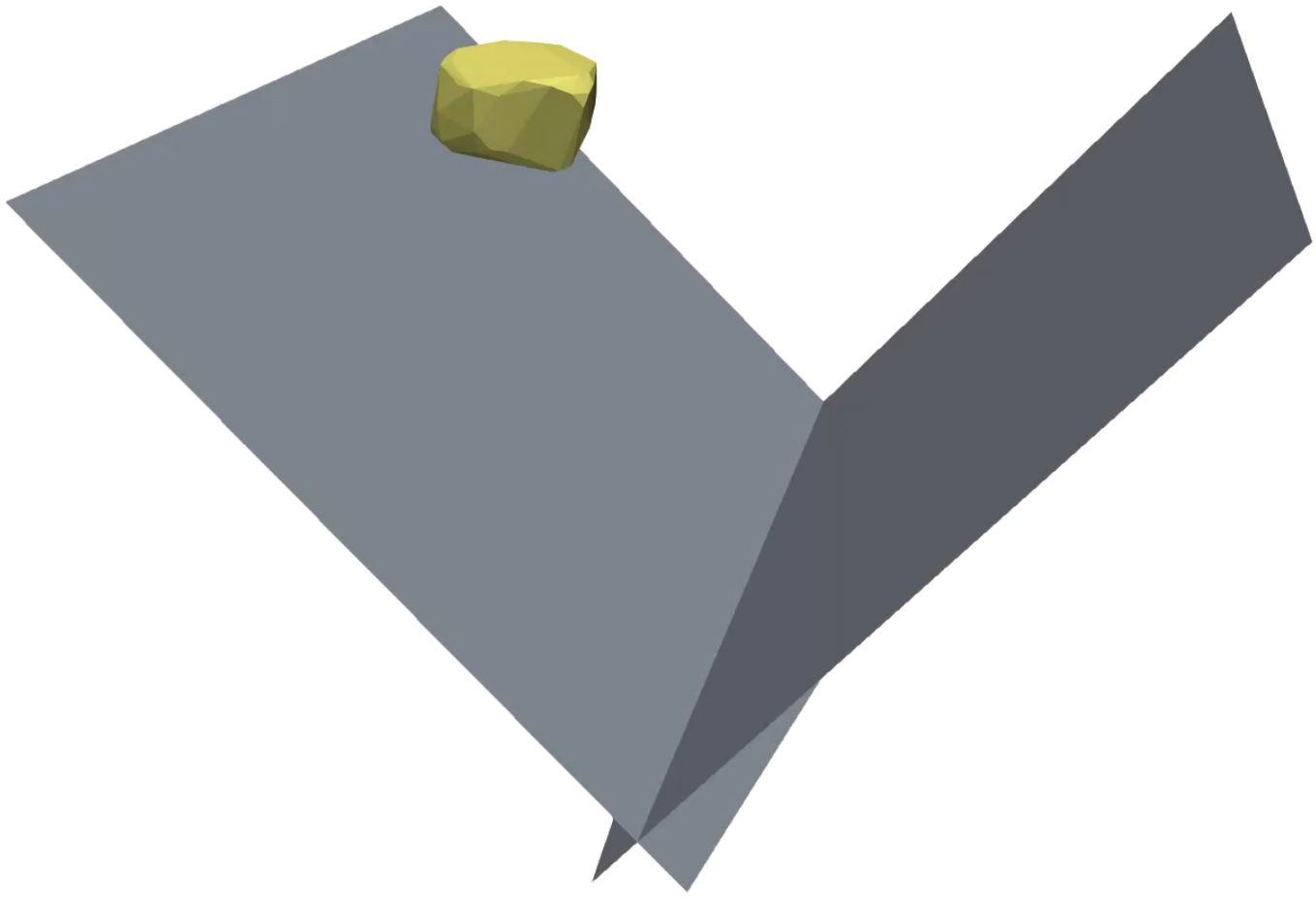
angular momentum eq. : ${}_K\boldsymbol{\Theta}_S\,{}_K\dot{\vec{\boldsymbol{\Omega}}} + {}_K\vec{\boldsymbol{\Omega}} \times ({}_K\boldsymbol{\Theta}_S\,{}_K\vec{\boldsymbol{\Omega}}) = {}_K\vec{\mathbf{M}}^a$ body fixed frame: K

$\mathbf{M}\dot{\mathbf{u}} - \mathbf{h}(\mathbf{q}, \mathbf{u}) = \mathbf{0}$ or $= \mathbf{W}\boldsymbol{\lambda}$ $\qquad \dot{\mathbf{q}} = \begin{pmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{H}(\mathbf{p}) \end{pmatrix} \mathbf{u}$

$\mathbf{M} = \begin{pmatrix} m\mathbf{I} & \mathbf{O} \\ \mathbf{O} & {}_K\boldsymbol{\Theta}_S \end{pmatrix}, \qquad \mathbf{h}(\mathbf{q}, \mathbf{u}) = \begin{pmatrix} {}_I\vec{\mathbf{F}}^a \\ {}_K\vec{\mathbf{M}}^a - {}_K\vec{\boldsymbol{\Omega}} \times ({}_K\boldsymbol{\Theta}_S\,{}_K\vec{\boldsymbol{\Omega}}) \end{pmatrix}$

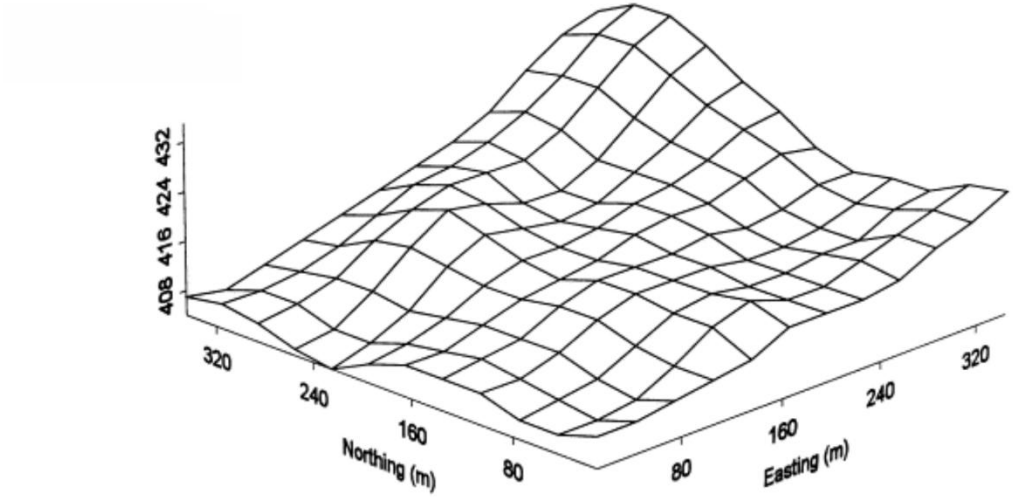
mass matrix is constant and invertible

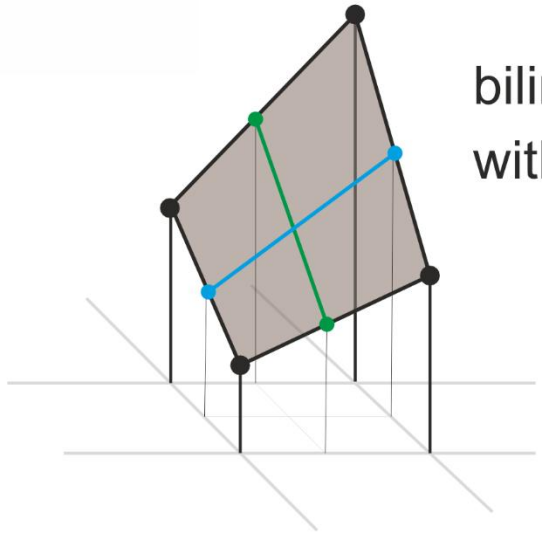
Rock on inclined half-planes



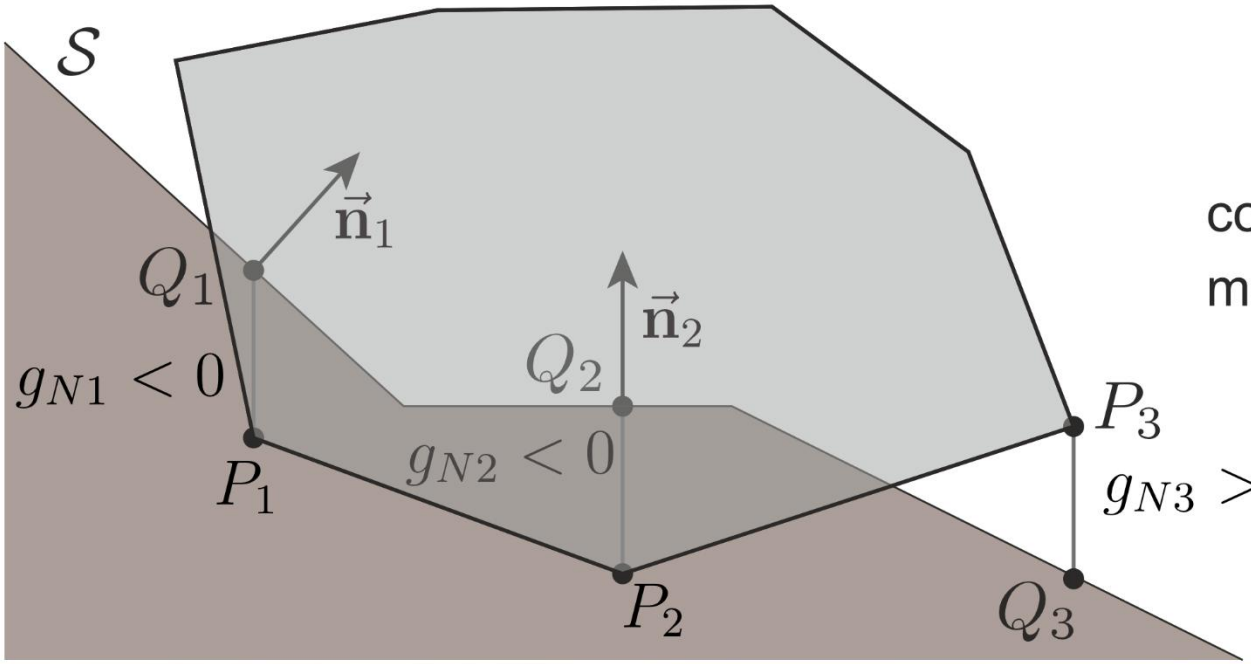
Terrain model

Digital Elevation Model (DEM)



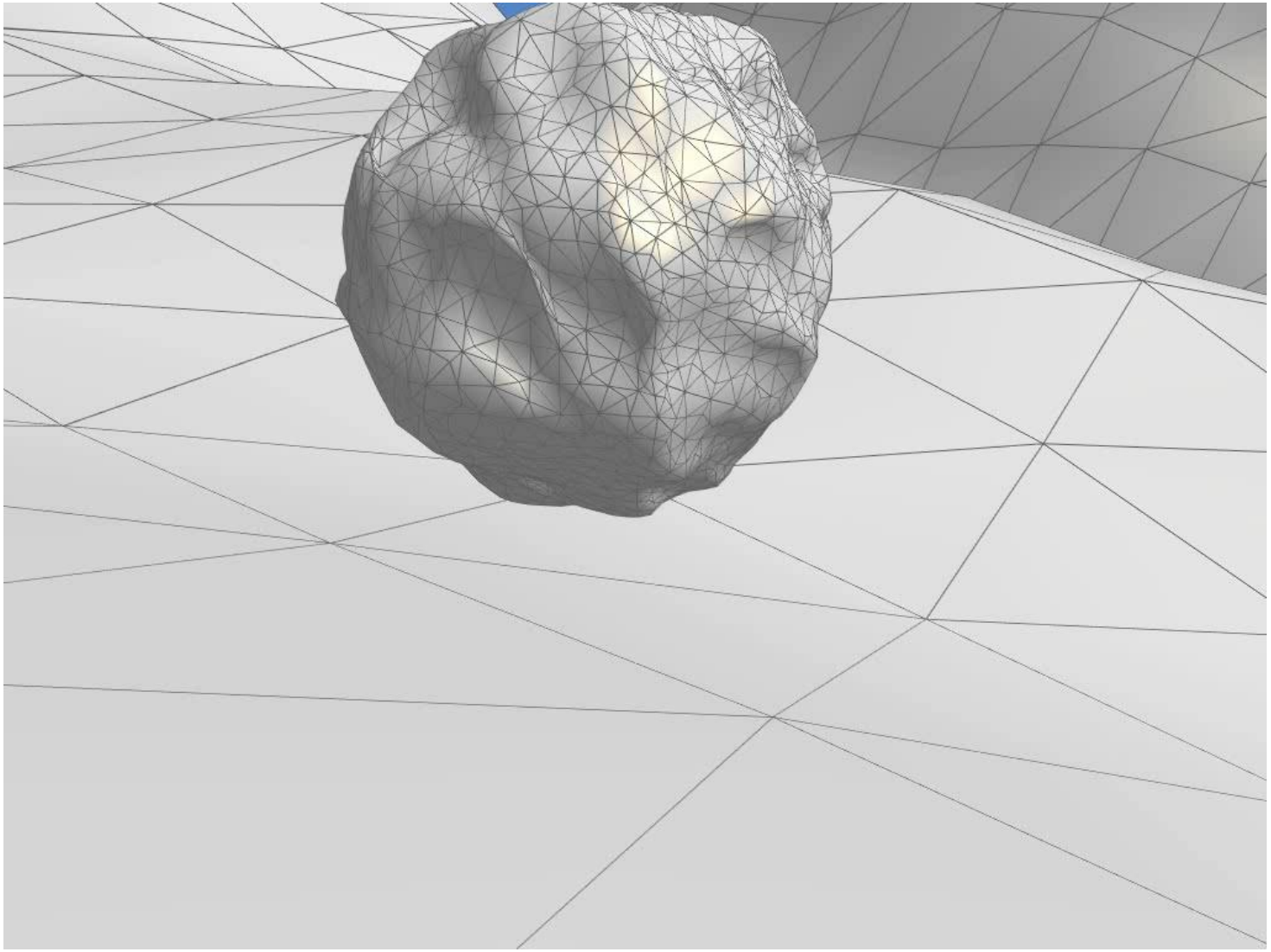


bilinear interpolation
within a grid cell



contact distance
measured vertically

Simulation (preliminary)



Adrian Schweizer & R.L., 2012



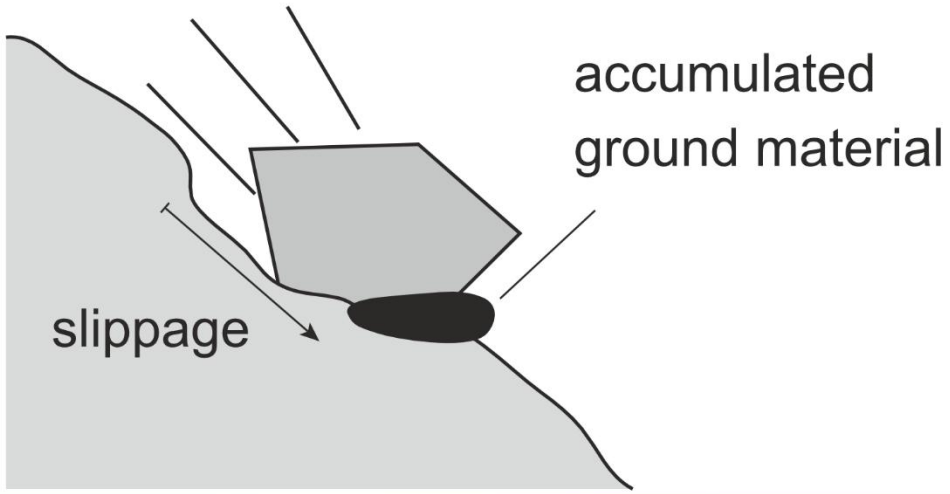
W. Gerber, WSL

restitution coefficient is zero!

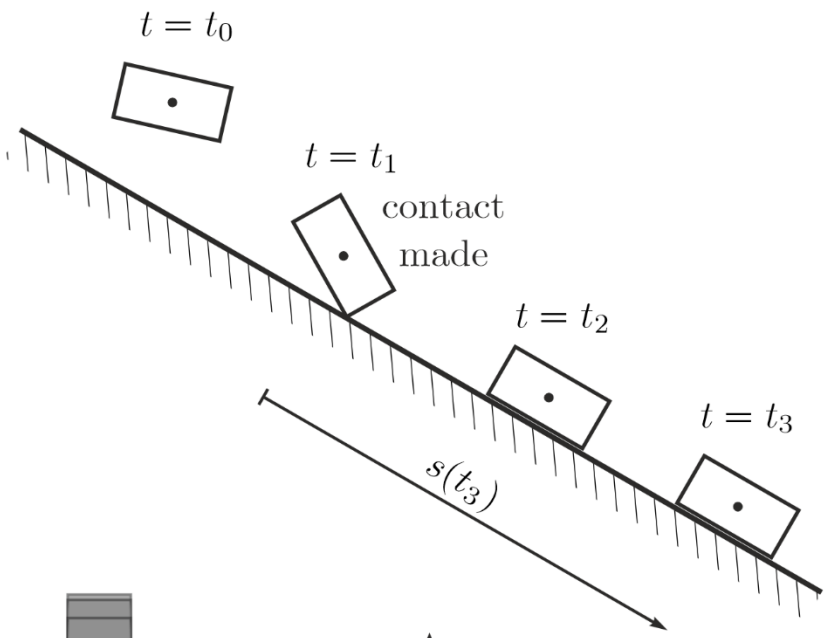
Contact model: scarring

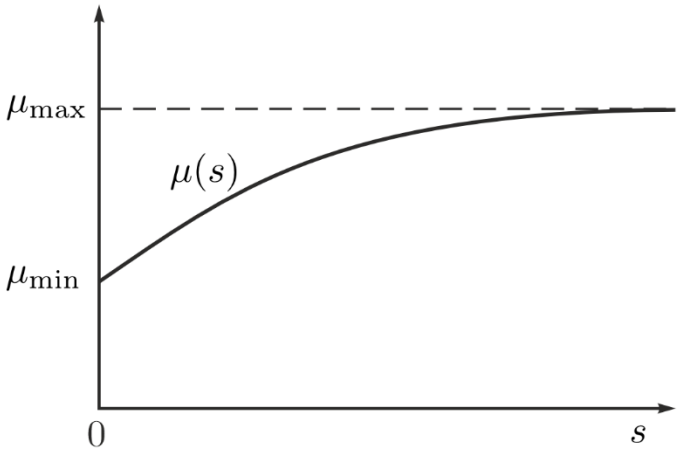


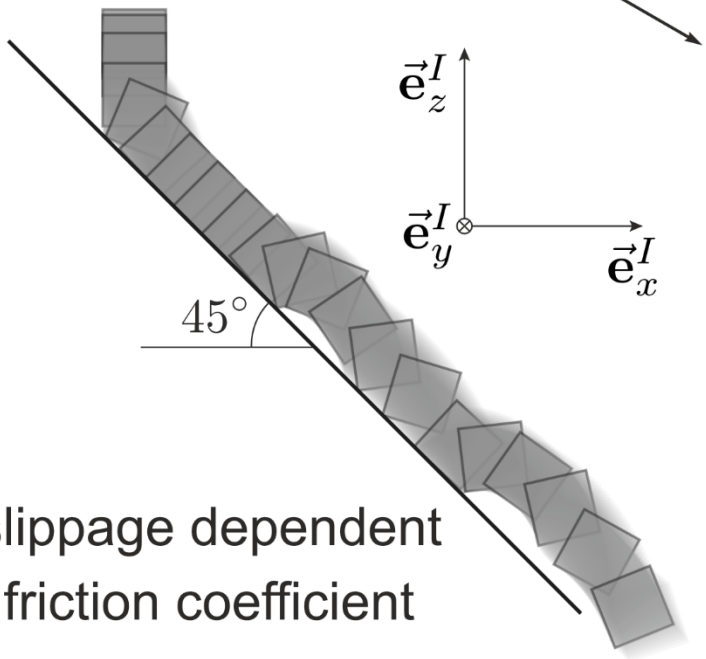




Contact model: slippage dependent friction

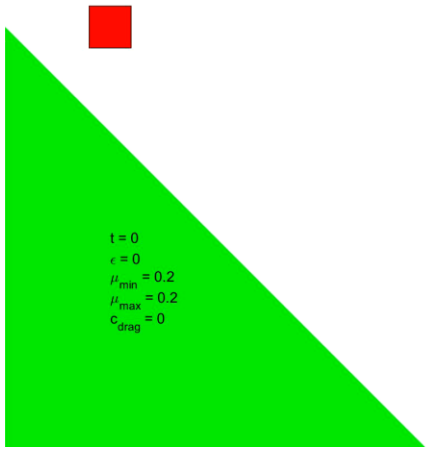




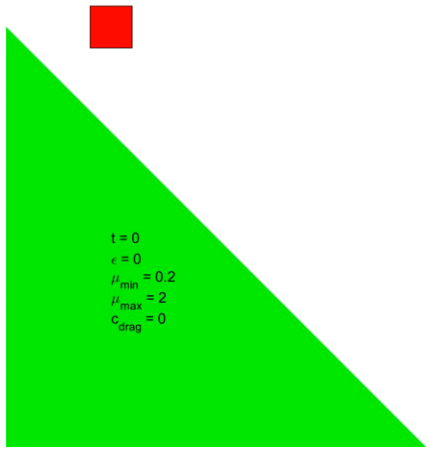


slippage dependent
friction coefficient

completely inelastic
standard Coulomb
friction model



completely inelastic
slippage dependent
friction model

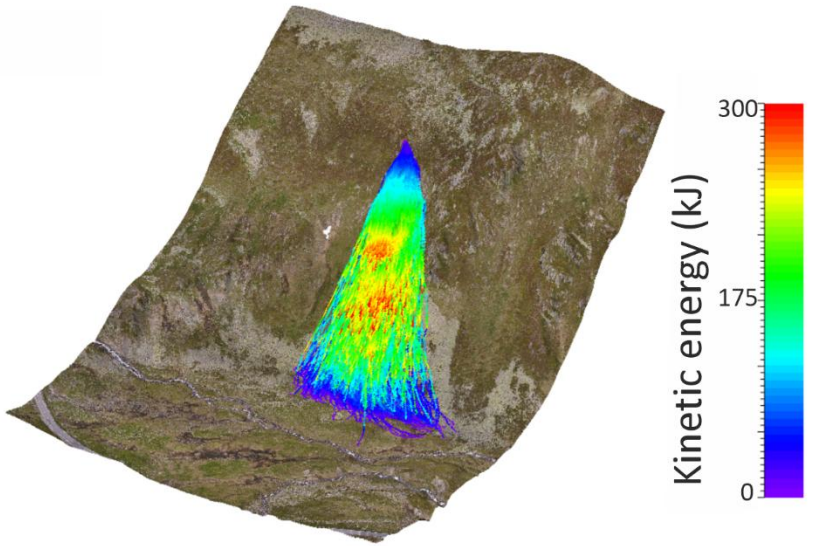


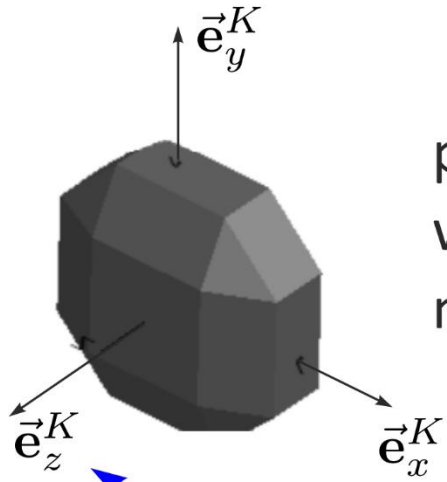
Rockfall field experiment at Flüelapass, Switzerland



courtesy of SLF

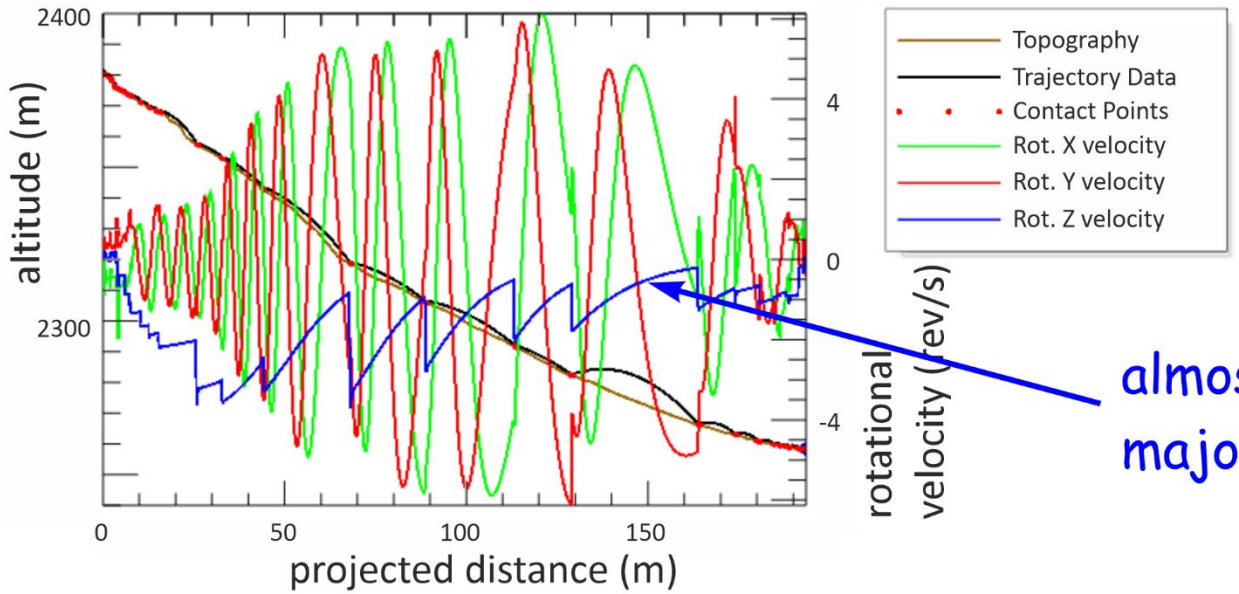
Rockfall simulation for Chant Sura (Davos)





platy boulder
with z-axis as
major principal axis

major principal axis



almost no rotation around
major principal axis

Rotation around principal axes



minor principal axis

stable



intermediate principal axis

unstable



major principal axis

stable

Euler equations $${}_K\boldsymbol{\Theta}_S\,{}_K\dot{\vec{\boldsymbol{\Omega}}} + {}_K\vec{\boldsymbol{\Omega}} \times ({}_K\boldsymbol{\Theta}_S\,{}_K\vec{\boldsymbol{\Omega}}) = \mathbf{0}$$

rot. kinetic energy $$T = \tfrac{1}{2}\,{}_K\vec{\boldsymbol{\Omega}}^{\mathrm{T}}\,{}_K\boldsymbol{\Theta}_S\,{}_K\vec{\boldsymbol{\Omega}}$$

angular momentum $$\|\vec{\mathbf{N}}_S\|^2 = {}_K\vec{\boldsymbol{\Omega}}^{\mathrm{T}}\,{}_K\boldsymbol{\Theta}_S^2\,{}_K\vec{\boldsymbol{\Omega}}$$

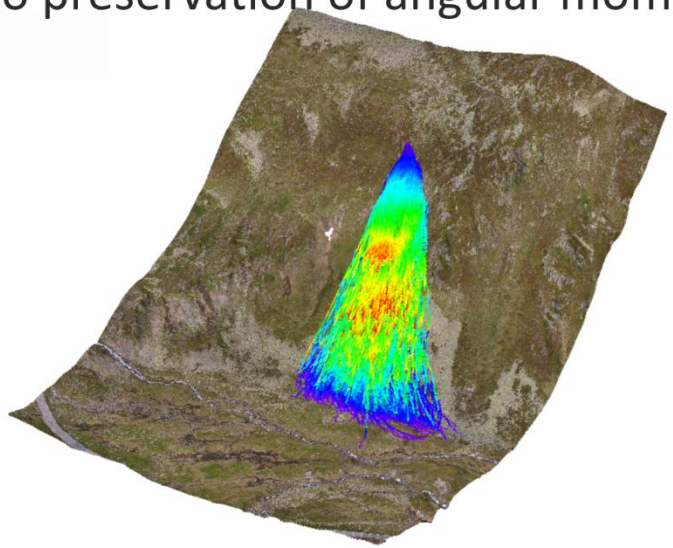
} invariants of motion

Stability properties follow from Lyapunov functions on these invariants

RAMMS::Rockfall Simulation for Chant Sura

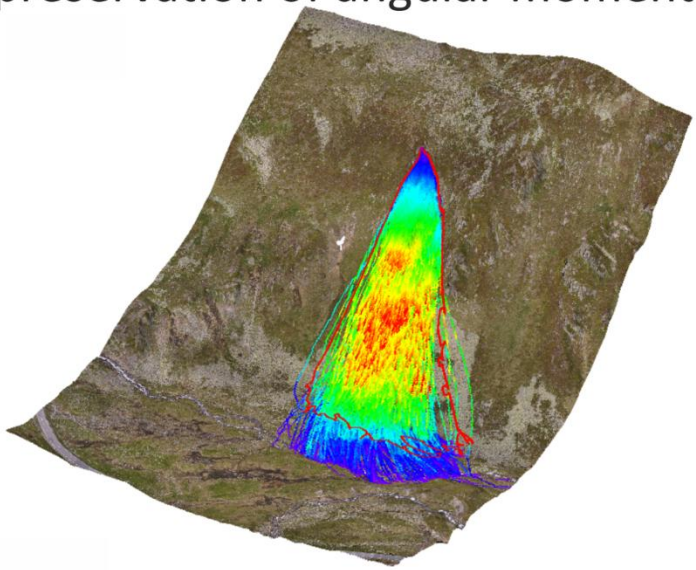


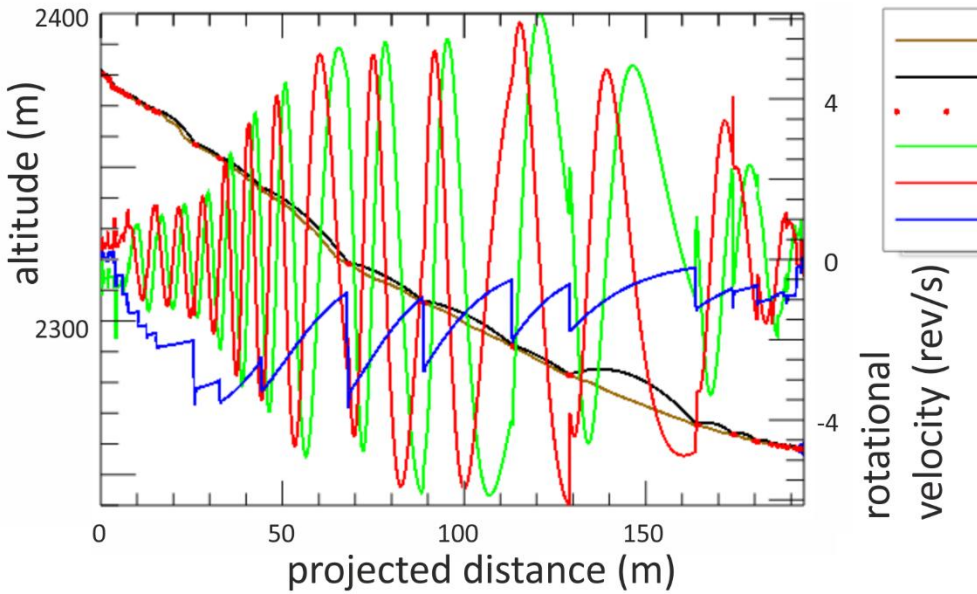
no preservation of angular momentum

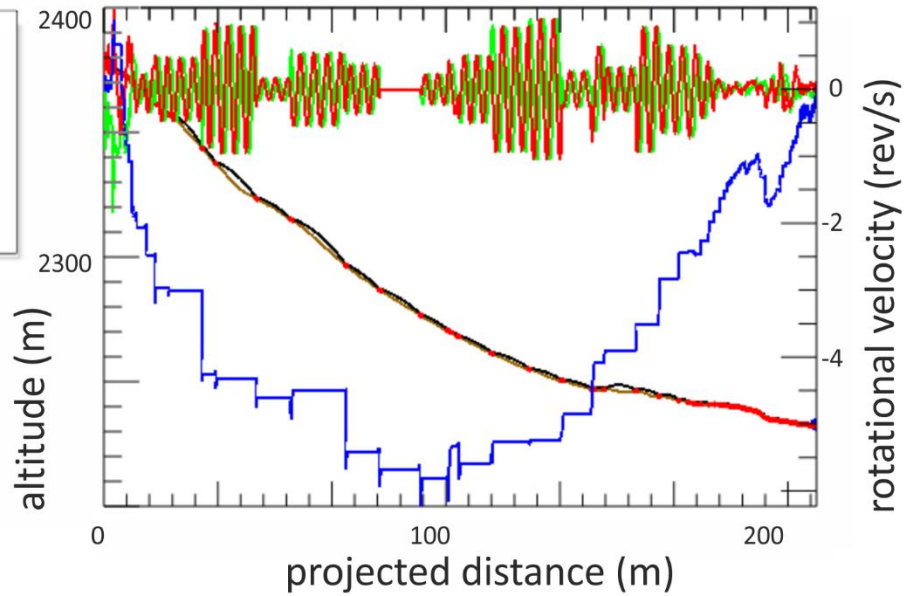




with preservation of angular momentum







Rockfall tests on Svalbard, August 2023





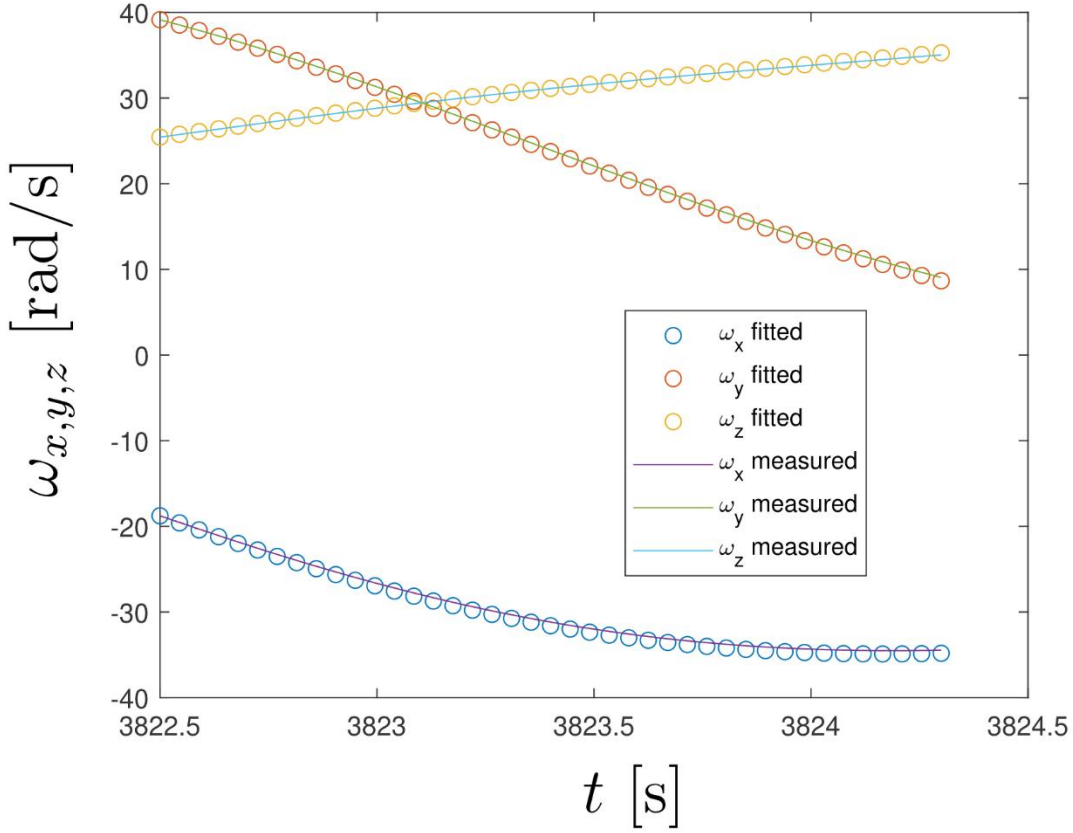





Rockfall tests on Svalbard, August 2023



Experiment during free flight with equant rock



least squares fit of

$$ {}_K\boldsymbol{\Theta}_S = \begin{pmatrix} A & D & E \\ D & B & F \\ E & F & C \end{pmatrix} $$

on Euler equations for a rigid free body

$$ {}_K\boldsymbol{\Theta}_S \, {}_K\dot{\vec{\Omega}}_{\text{meas}} + {}_K\vec{\Omega}_{\text{meas}} \times \left({}_K\boldsymbol{\Theta}_S \, {}_K\vec{\Omega}_{\text{meas}} \right) = \mathbf{0} $$

$$ {}_K\boldsymbol{\Theta}_S = A \begin{pmatrix} 1.0000 & 0.0056 & -0.0082 \\ 0.0056 & 1.0058 & 0.0094 \\ -0.0082 & 0.0094 & 1.0075 \end{pmatrix} $$

$\Rightarrow$ ode45 to find ${}_K\vec{\Omega}_{\text{fitted}}$

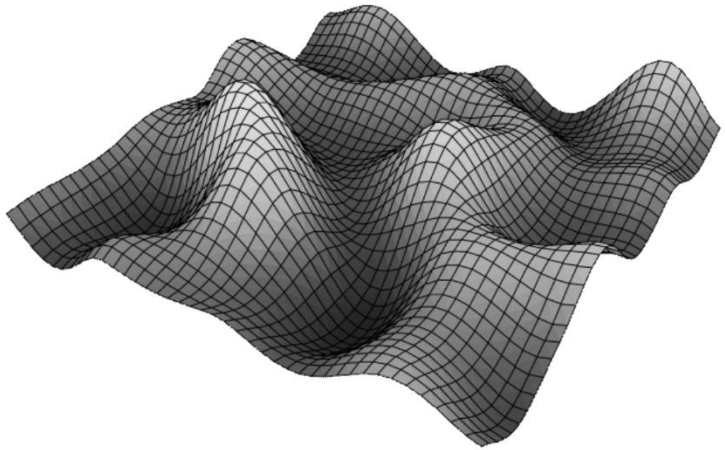
Towards a scree model: roughness



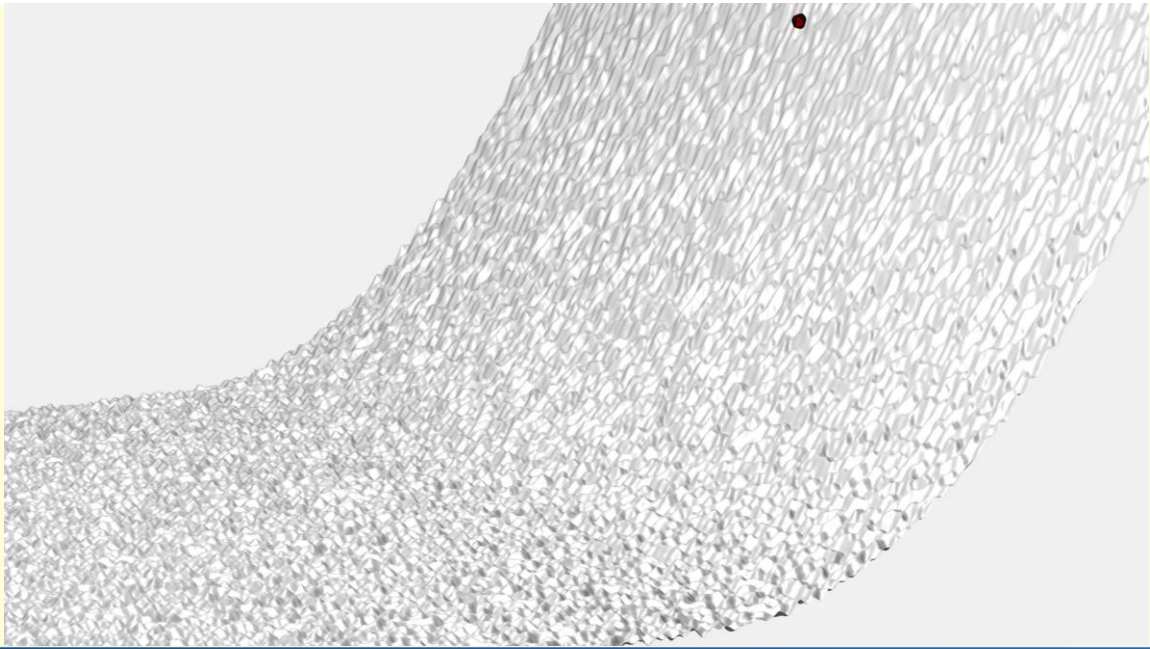
Scree



3D Perlin noise



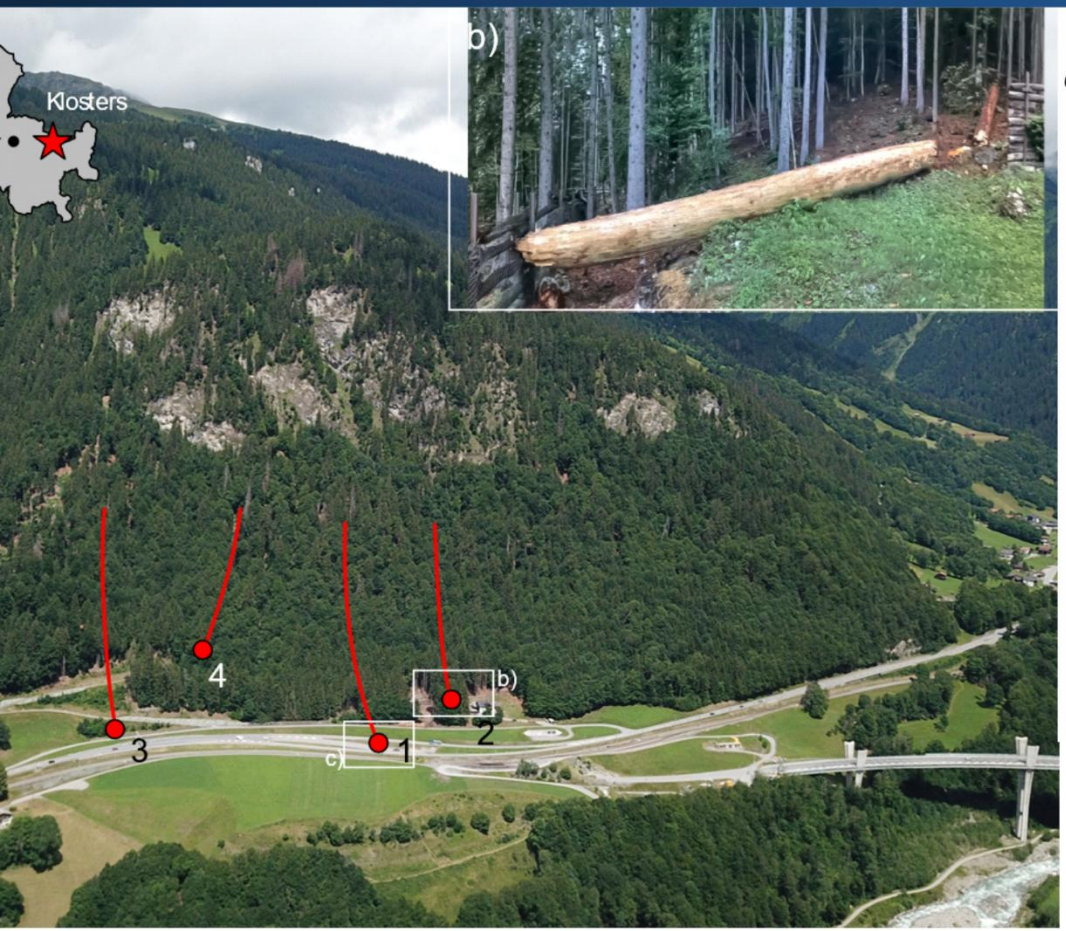
roughness enhancement of Digital Elevation Model using Perlin noise

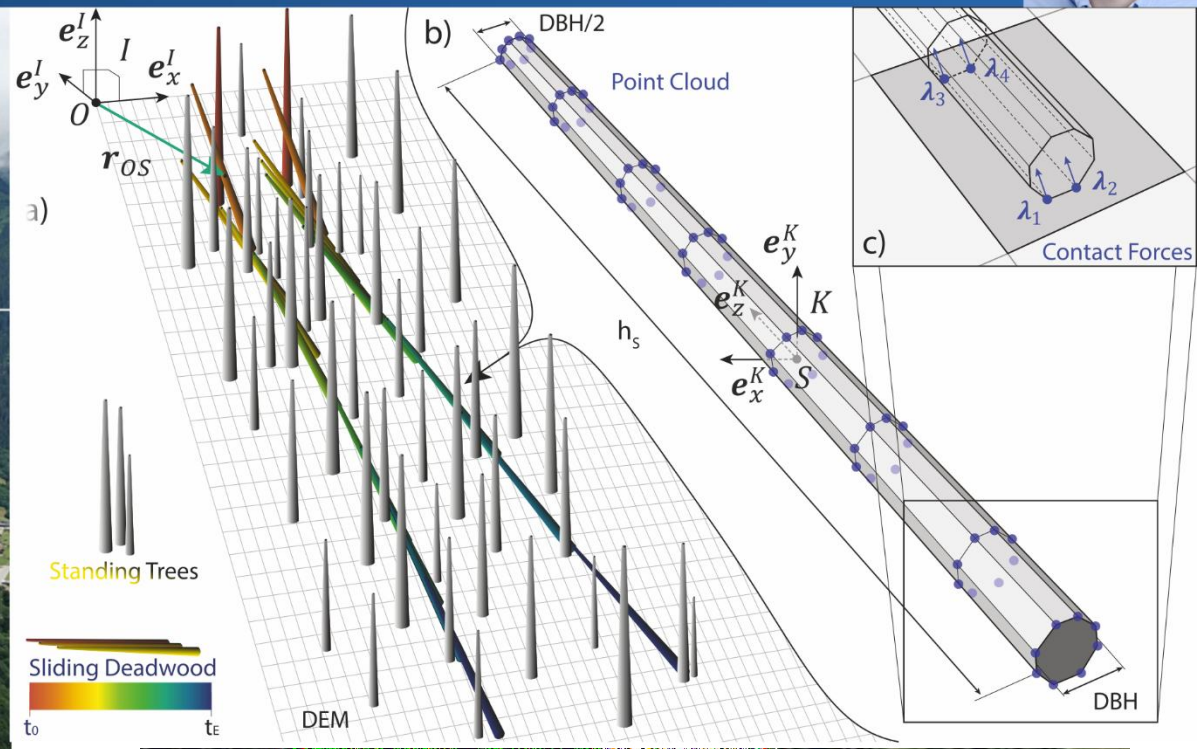


Simulation by **Joel Borner**

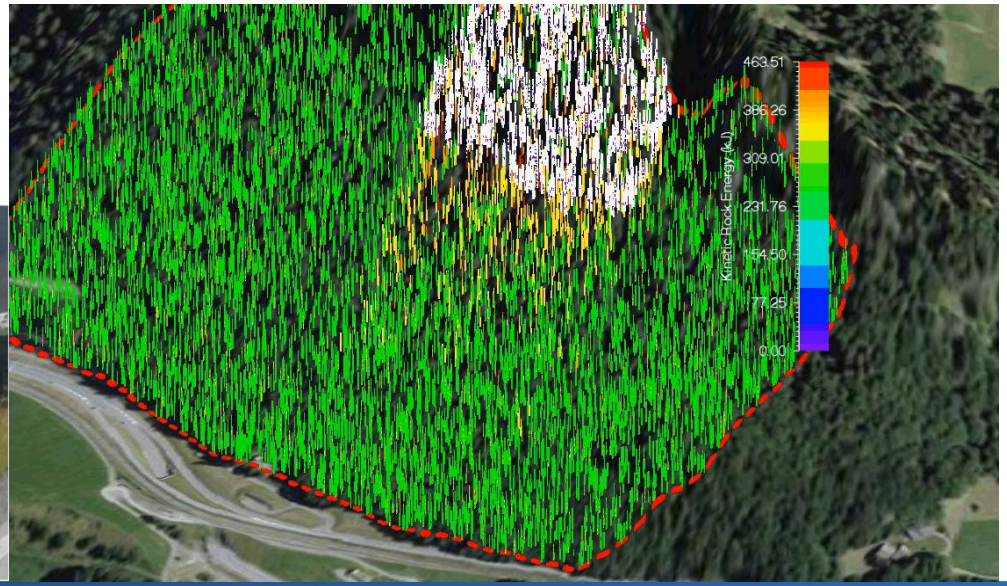
Sliding deadwood



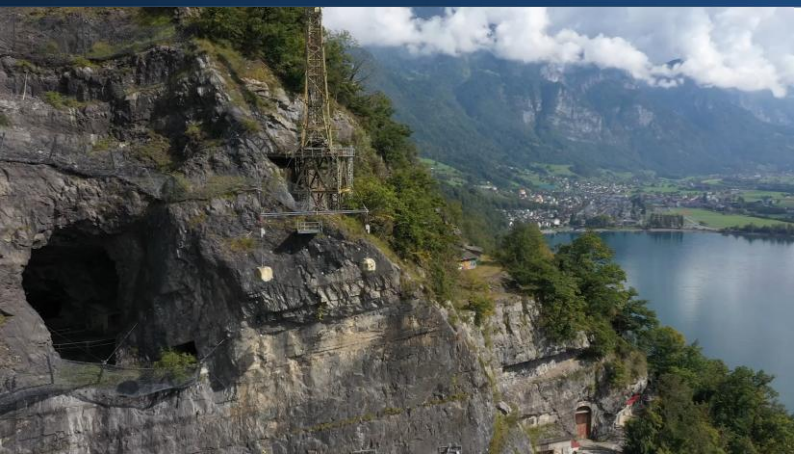






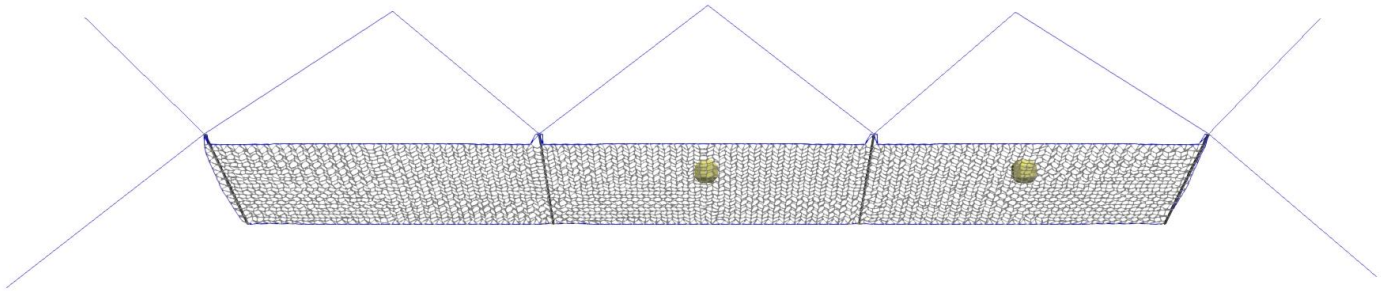


Simulation of rockfall protection nets

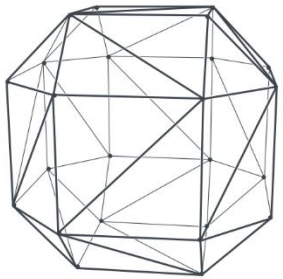


Full scale experiment (GEOBRUGG)

Simulation by Lisa Eberhardt

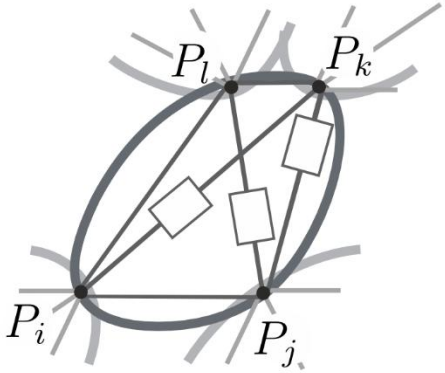


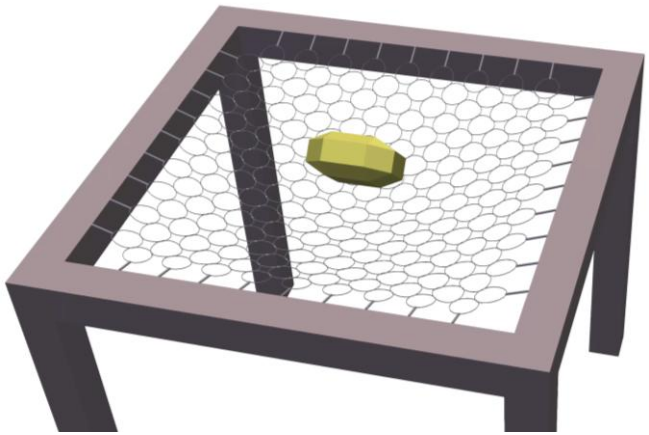
Rock model





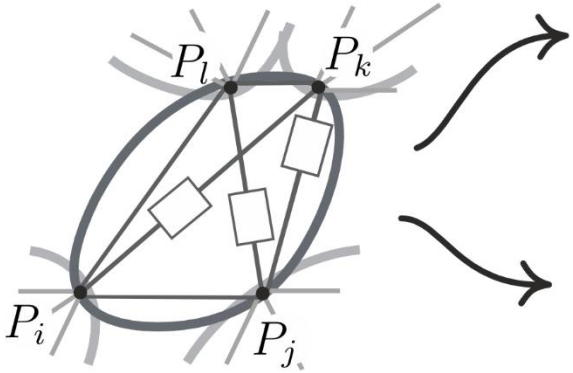
Net model

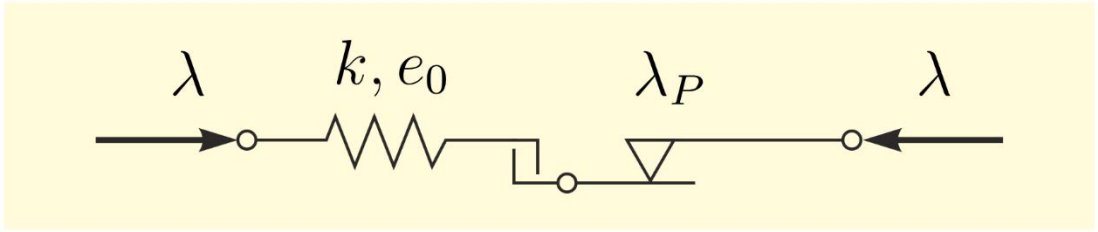


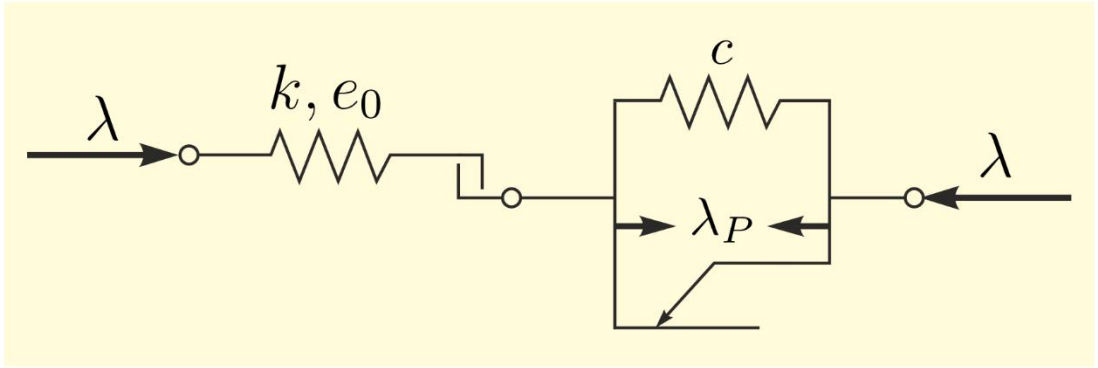


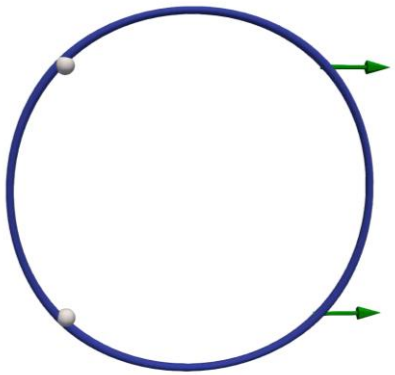
Simulation of rockfall protection nets

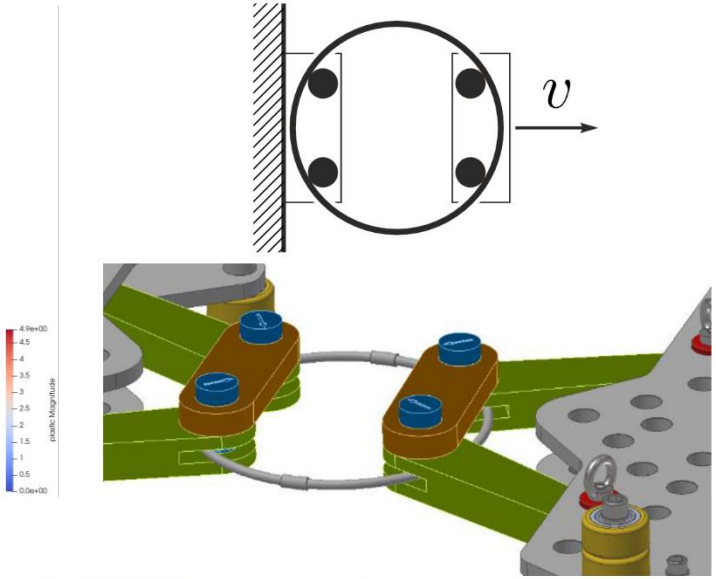


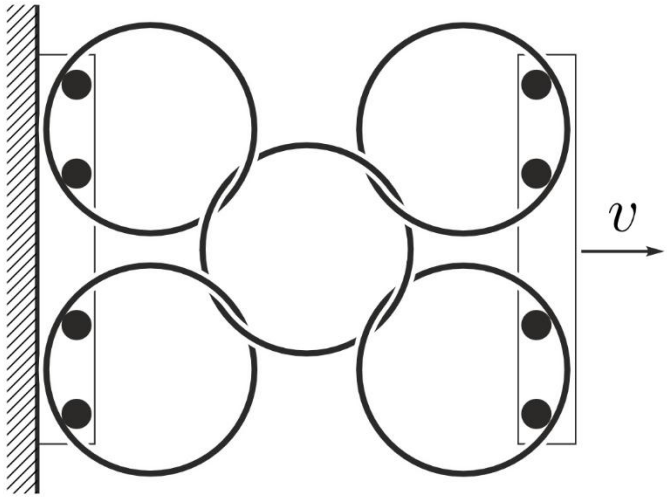












Conclusions

- **Rock shape** is of prime importance for rockfall hazard mapping and mitigation
- **Rockfall research benefits from**

a) **Nonlinear Dynamics**: e.g. stability properties of rotating rocks

Open Question: How to show stability of the major principal axis for a rock bouncing down the slope?

b) **Nonsmooth Dynamics**: e.g. set-valued constitutive laws, simulation methods

Open Question: How to formulate advanced contact models for rock terrain interaction?



Lisa Eberhardt (Uni Stuttgart)



Joël Borner (SLF)



Adrian Ringenbach (ETH)



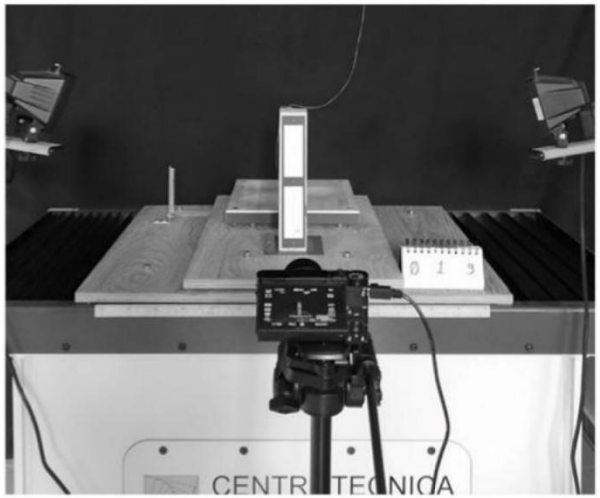
Jonas Breuling (Uni Stuttgart)



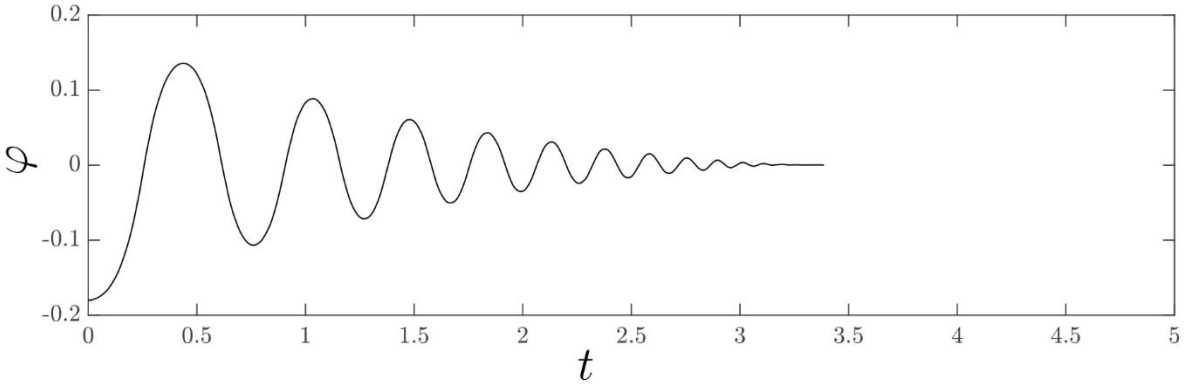
Giuseppe Capobianco (FAU Erlangen)

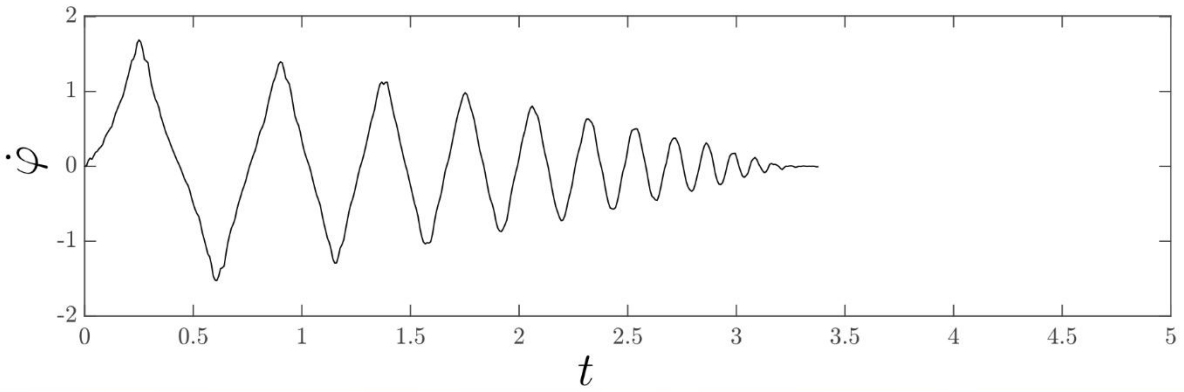
With many thanks to some of my PhD students and co-workers

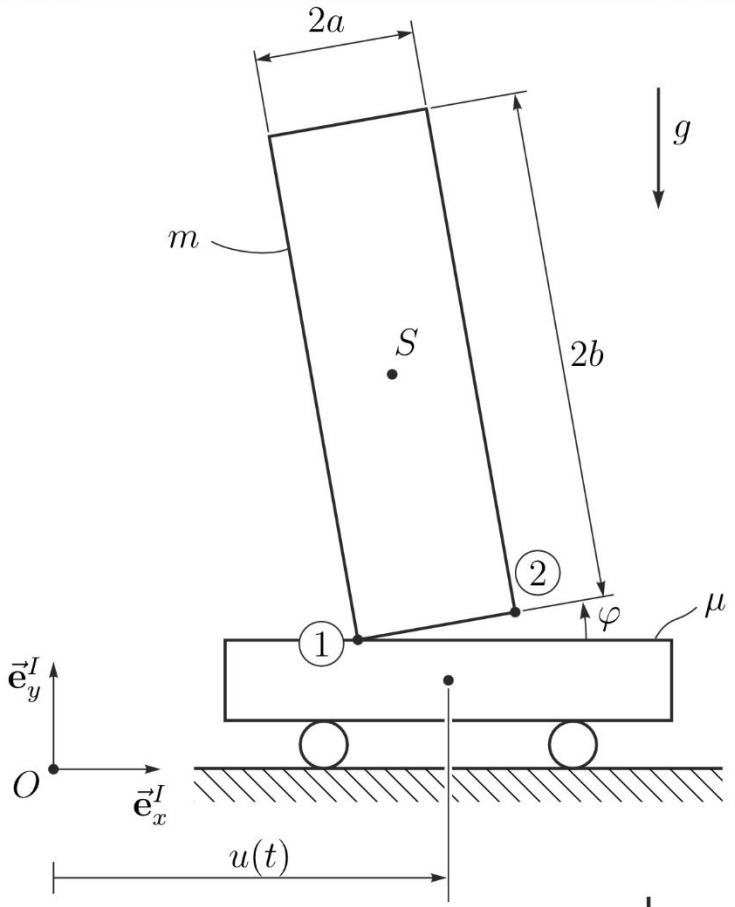
Rocking block experiment of Cocuzza Avelino 2022

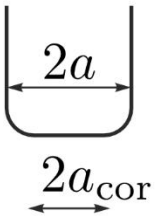












Uplift acceleration:

$$\ddot{u}_{\text{exp}} = 0.240g$$

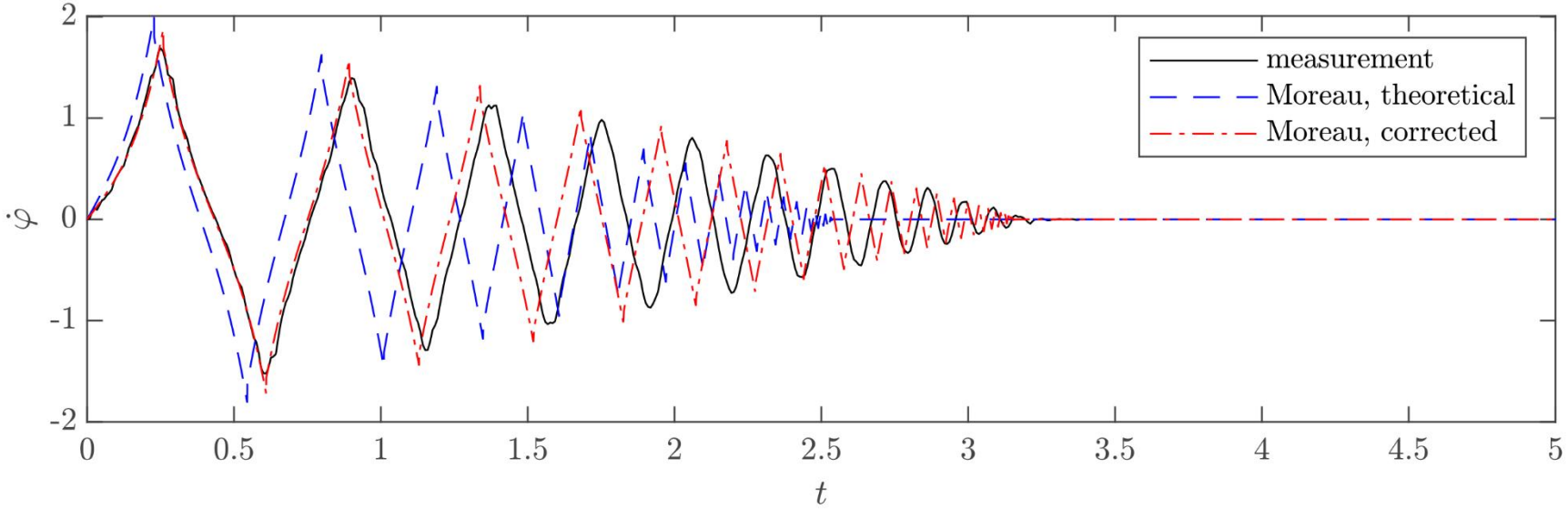
$$\ddot{u}_{\text{theor}} = \frac{a}{b}g = 0.264g$$

Corrected width $a_{\text{cor}} = \frac{\ddot{u}_{\text{exp}}}{g}b$

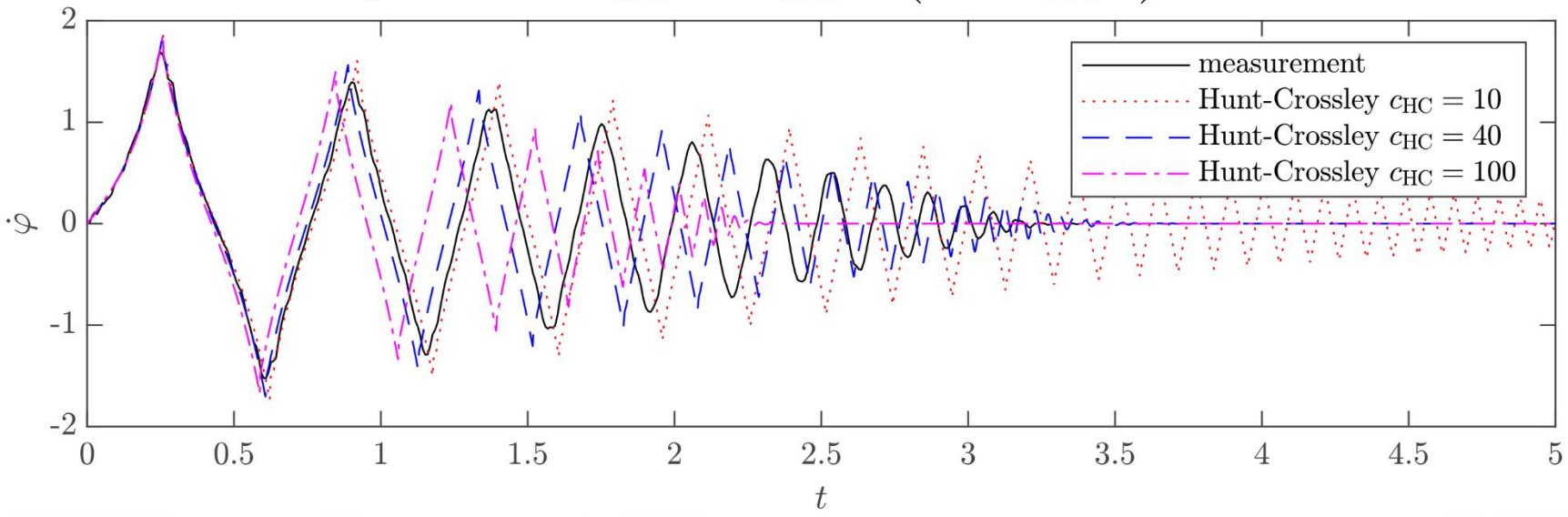
Cocuzza Avelino et al, "Shaking table seismic experimental ..", Buildings, 2022

Rocking block: comparison of numerical methods

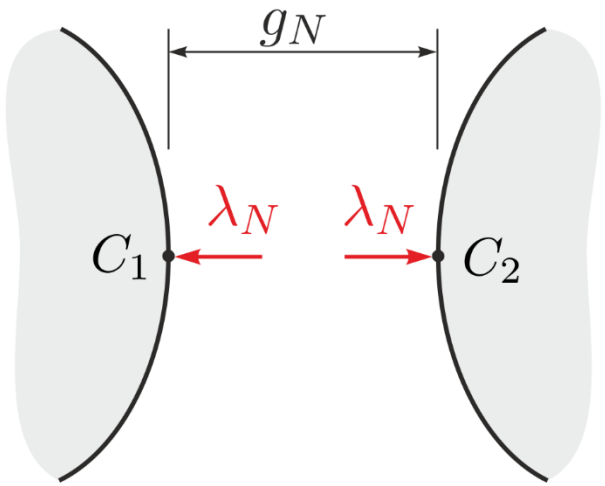
Moreau timestepping with $e_N = 0$



Hunt Crossley model $F_{\mathrm{HC}} = k_{\mathrm{HC}}\delta^{\frac{3}{2}}(1 + c_{\mathrm{HC}}\dot{\delta})$



Contact law in normal direction



Signorini's law

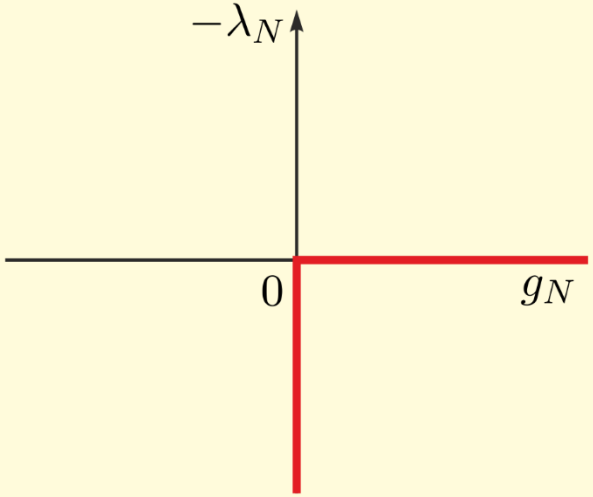
no penetration $g_N \geq 0$

no adhesion $\lambda_N \geq 0$

no distance effects $g_N > 0 \Longrightarrow \lambda_N = 0$

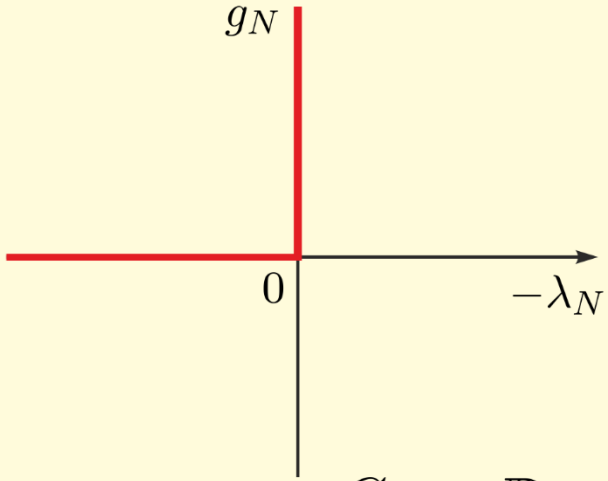
$$0 \leq g_N \perp \lambda_N \geq 0$$

Primal form $-\lambda_N \in N_{\mathbb{R}_0^+}(g_N)$



$\Longleftrightarrow$

Dual form $g_N \in N_{C_N}(-\lambda_N)$



force reservoir: $C_N = \mathbb{R}_0^-$

Dynamics: the evolutionary problem

Nonimpulsive dynamics

kinematics: $\dot{\boldsymbol{q}} = \boldsymbol{u}$

equation of motion: $\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{u}} - \boldsymbol{h}(\boldsymbol{q}, \boldsymbol{u}) = \boldsymbol{W}_N \boldsymbol{\lambda}_N + \boldsymbol{W}_T \boldsymbol{\lambda}_T$

normal contact law: $g_{Nj} \in N_{C_N}(-\lambda_{Nj})$

friction law: $\gamma_{Tj} \in N_{C_T(\lambda_{Nj})}(-\lambda_{Tj})$

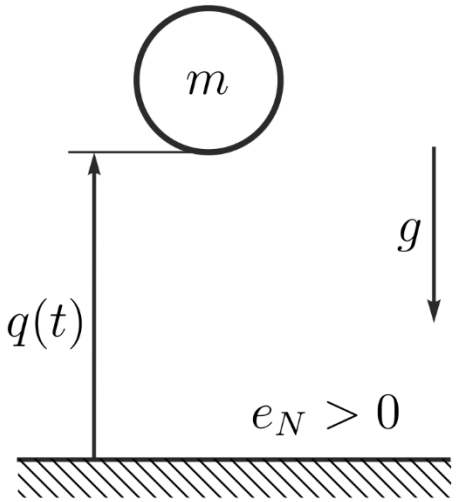
Impulsive dynamics

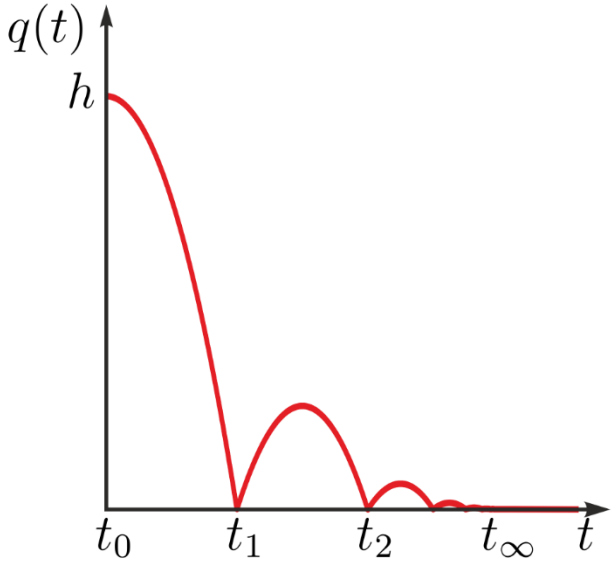
impact equation: $\boldsymbol{M}(\boldsymbol{q})(\boldsymbol{u}^+ - \boldsymbol{u}^-) = \boldsymbol{W}_N \boldsymbol{\Lambda}_N + \boldsymbol{W}_T \boldsymbol{\Lambda}_T$

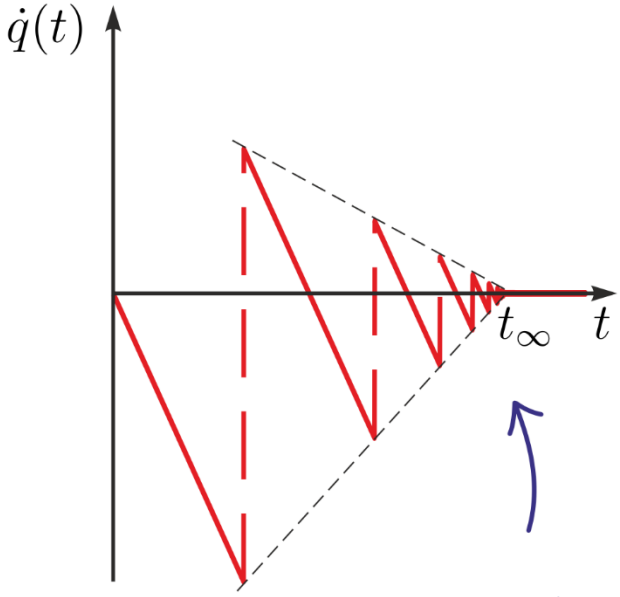
normal impact law: $\gamma_{Nj}^+ + e_{Nj}\gamma_{Nj}^- \in N_{C_N}(-\Lambda_{Nj}), \quad j \in \mathcal{I}_N = \{j \mid g_{Nj} = 0\}$

tangential impact law: $\gamma_{Tj}^+ + e_{Tj}\gamma_{Tj}^+ \in N_{C_T(\Lambda_{Nj})}(-\Lambda_{Tj}), \quad j \in \mathcal{I}_N$

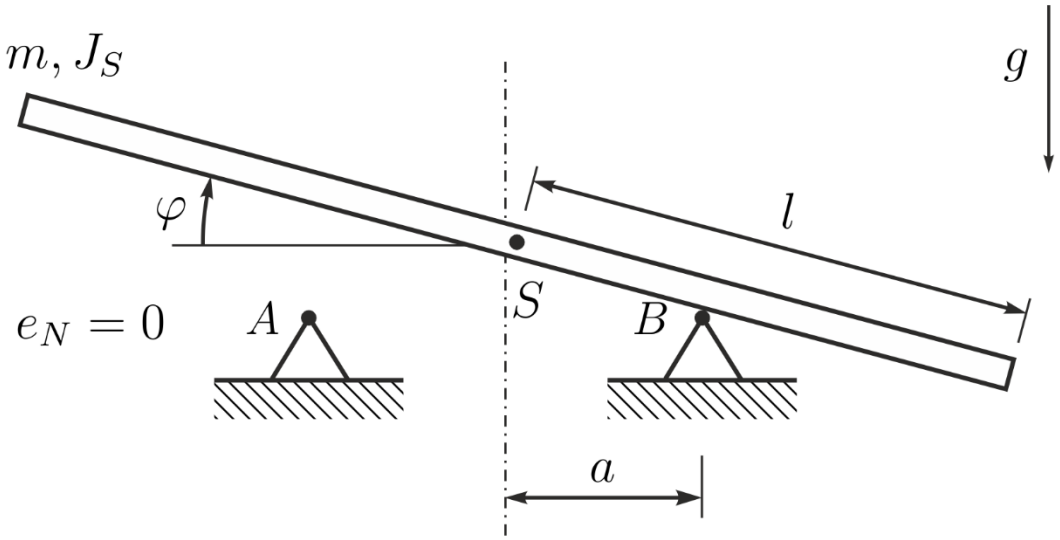
Accumulation points of collisions

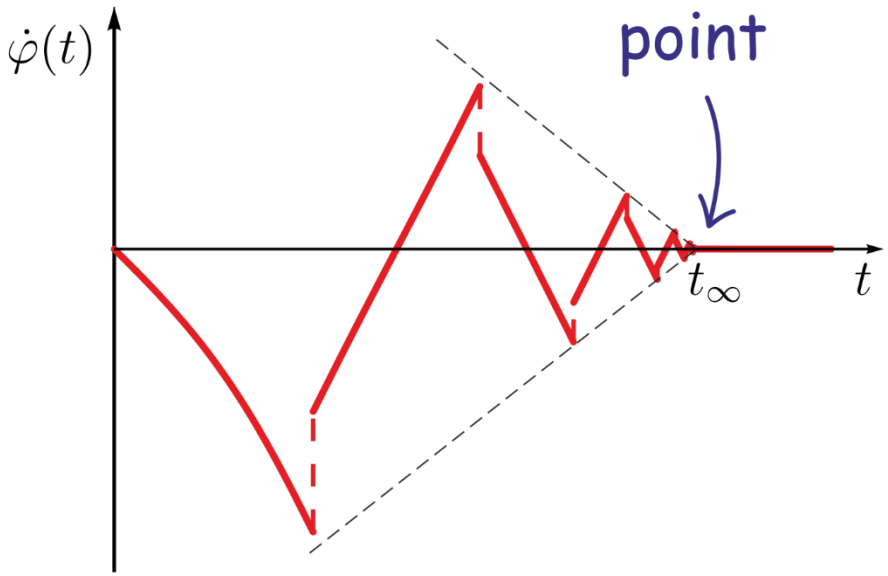






accumulation point





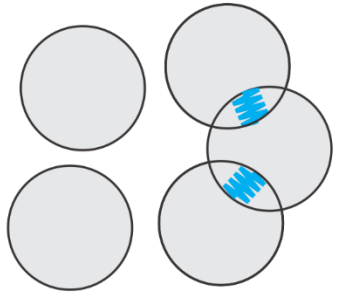
Discrete Element Methods

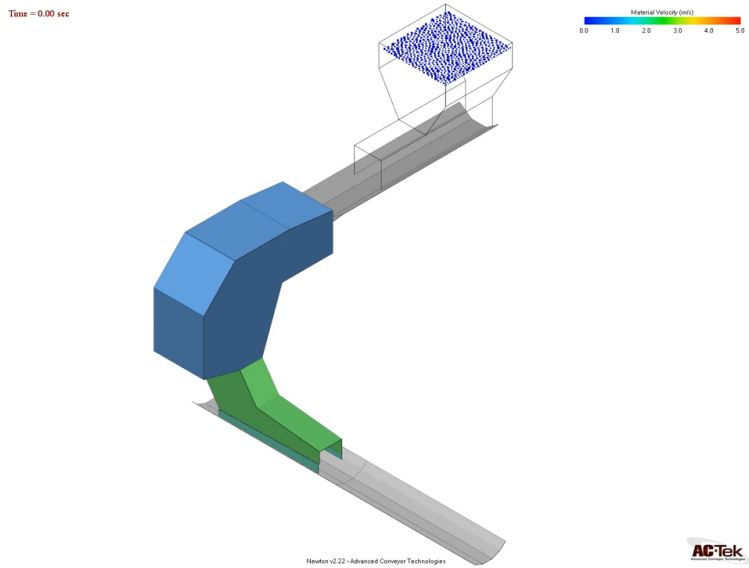
Common DEM / Molecular Dynamics

DEM with soft contact

explicit scheme

Cundall 1971





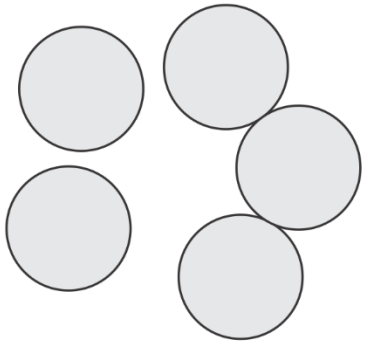
D. Kruse, 2014

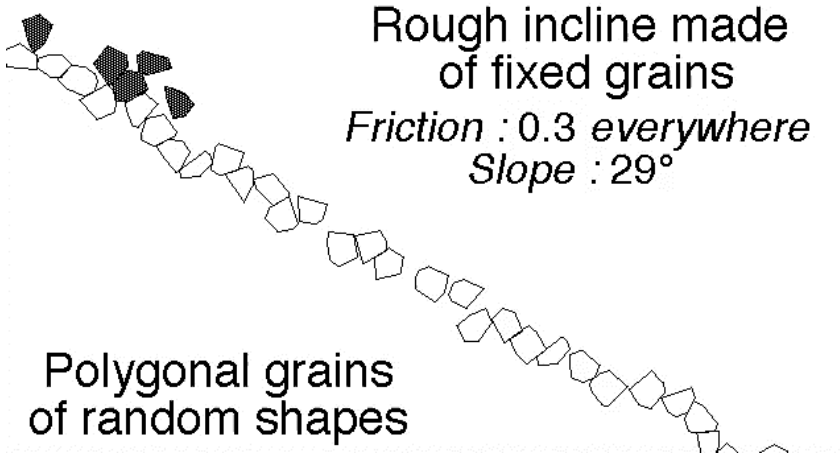
Nonsmooth Contact Dynamics

DEM with hard contact

implicit scheme

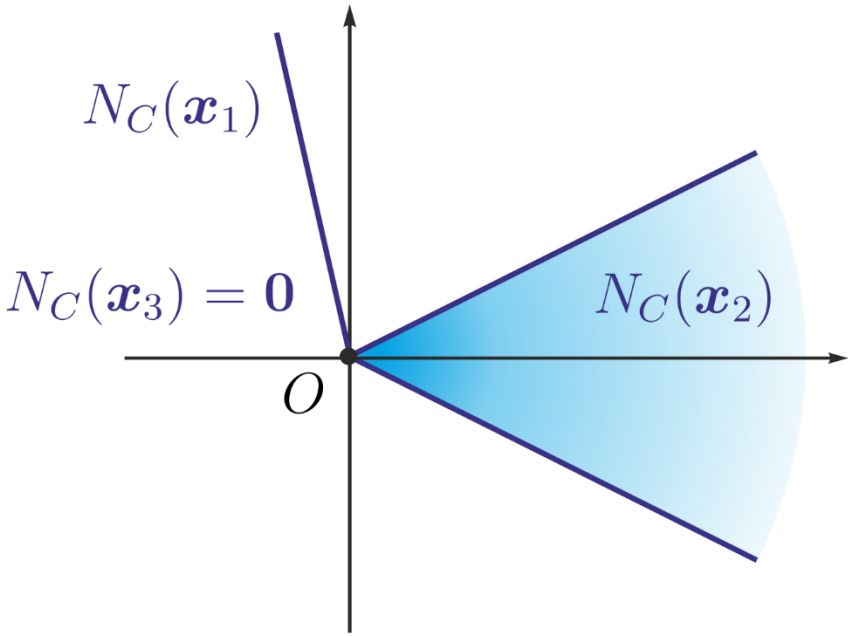
Moreau 1988

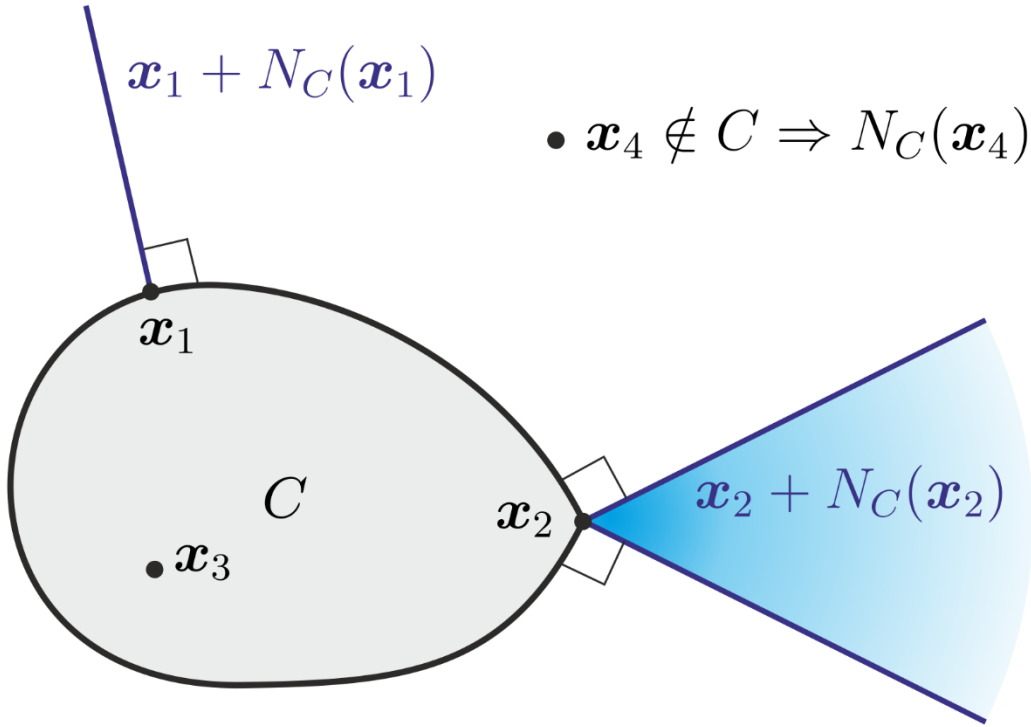




J.J. Moreau, 2001

Normal cone from convex analysis



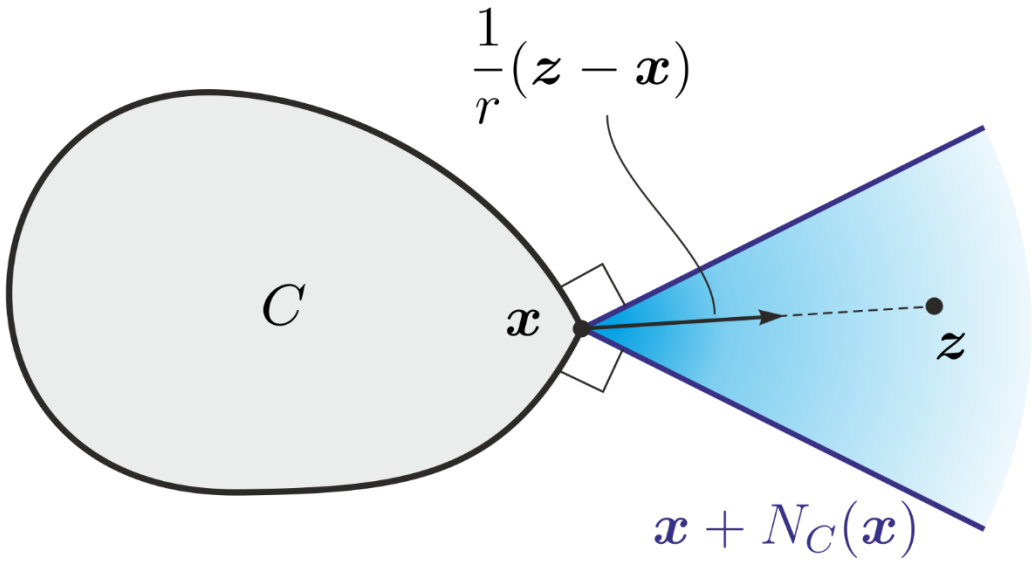


- $\boldsymbol{x}_4 \notin C \Rightarrow N_C(\boldsymbol{x}_4) = \emptyset$

$$N_C(\boldsymbol{x}) = \{\boldsymbol{y} \mid \boldsymbol{y}^{\mathrm{T}}(\boldsymbol{x}^* - \boldsymbol{x}) \leq 0,\ \boldsymbol{x} \in C, \forall \boldsymbol{x}^* \in C\}$$

$\boldsymbol{x}$ on the boundary of C	$\Rightarrow$	normal cone $N_C(\boldsymbol{x})$ consists of outward normal ray(s)
$\boldsymbol{x}$ in the interior of C	$\Rightarrow$	normal cone $N_C(\boldsymbol{x})$ is zero

Proximal point to a convex set



proximal point function:

$$\boldsymbol{x} = \text{prox}_C(\boldsymbol{z})$$

$$\Updownarrow$$

$$\frac{1}{r}(\boldsymbol{z}-\boldsymbol{x}) \in N_C(\boldsymbol{x}), \quad r>0$$

substitute:

$$\left.\begin{aligned} \boldsymbol{z} &= -\boldsymbol{\lambda} + r\boldsymbol{\gamma} \\ \boldsymbol{x} &= -\boldsymbol{\lambda} \end{aligned}\right\} \Rightarrow \begin{aligned} \boldsymbol{\gamma} &= \frac{1}{r}(\boldsymbol{z}-\boldsymbol{x}) \\ \boldsymbol{\lambda} &= -\boldsymbol{x} \end{aligned}$$

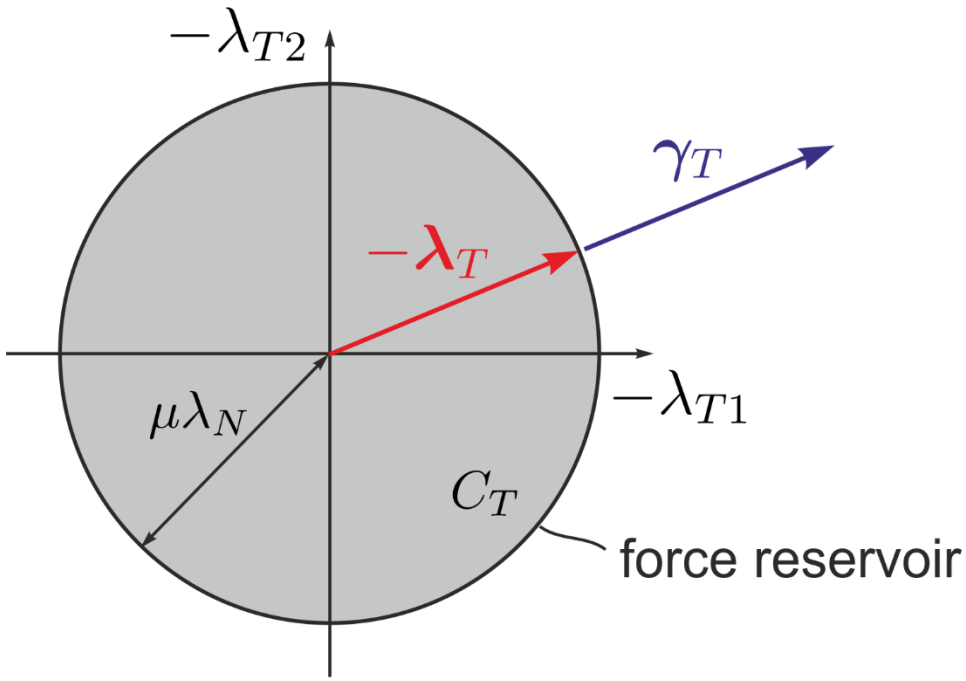
$$\boldsymbol{\gamma} \in N_C(-\boldsymbol{\lambda}) \quad \Longleftrightarrow \quad -\boldsymbol{\lambda} = \text{prox}_C(-\boldsymbol{\lambda} + r\boldsymbol{\gamma}) \qquad r>0$$

normal cone inclusion

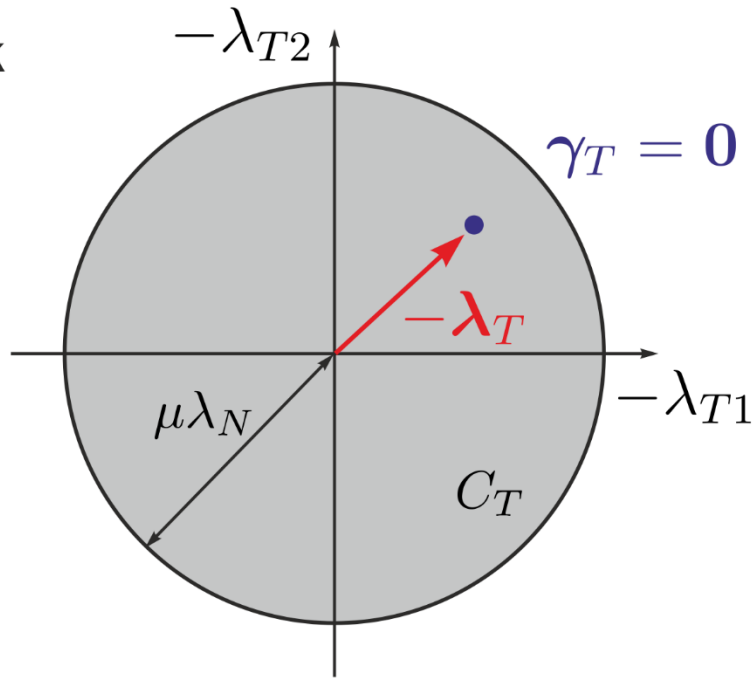
implicit equation

Coulomb's spatial friction law

slip



stick



$-\boldsymbol{\lambda}_T$ on the boundary of C_T

$\boldsymbol{\gamma}_T$ in outward normal ray

$-\boldsymbol{\lambda}_T$ in the interior of C_T

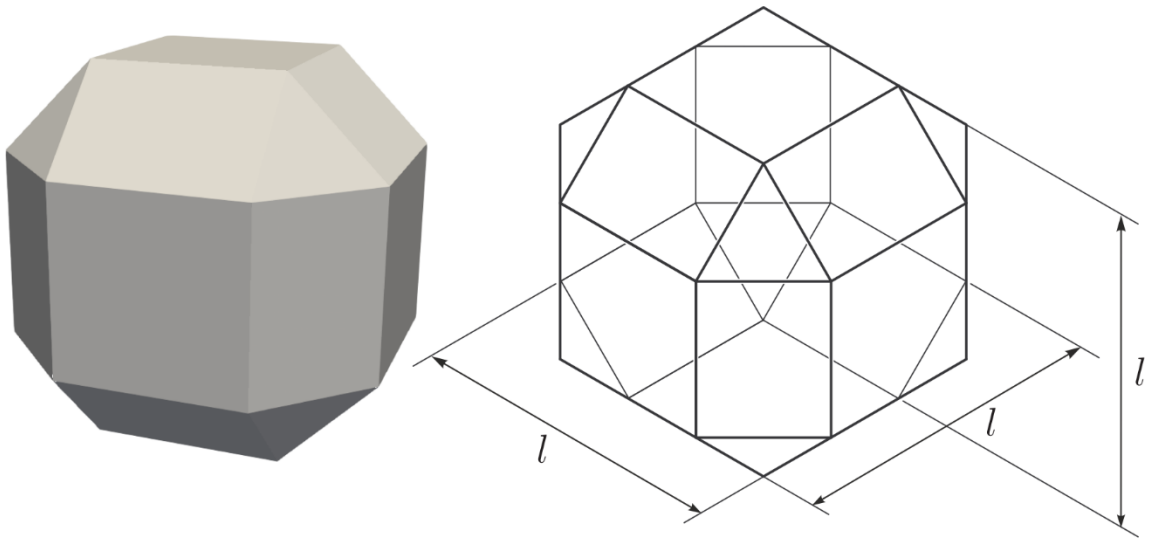
$\boldsymbol{\gamma}_T = \mathbf{0}$

$$\boldsymbol{\gamma}_T \in N_{C_T}(-\boldsymbol{\lambda}_T)$$

normal cone inclusion

Rock model: norm blocks

$EOTA_{111}$ norm block



$EOTA_{221}$ norm block

