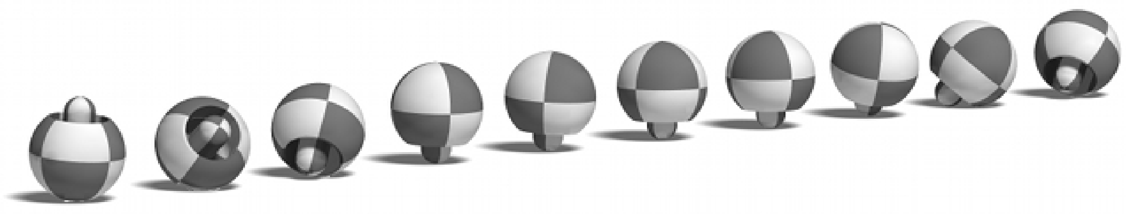
# Seventh Symposium of the
# European Network for Nonsmooth Dynamics (ENNSD)
# September 12-13, 2018, University of Stuttgart, Germany

Organized by: Remco Leine, Vincent Acary, and Olivier Brüls



This is the tentative program of March 2018, which may be subject to changes.

Institute for
Nonlinear Mechanics

## Program September, 12th

**08:30-08:50: Welcome, coffee**

**08:50-09:00: Opening**

**Session 1:**

**09:00-10:00**
Formulation and discretization of non-penetration constraints and impact laws
Presented by Olivier Brüls (University of Liège)

**10:00-11:00**
How does a non-smooth absorber work under near-resonant forcing?
Presented by Malte Krack (University of Stuttgart)

**11:00-11:30: Break**

**11:30-12:30**
Dynamics near periodic grazing orbits for vibro-impact systems through the first return time to the contact surface
Presented by Stephane Junca (University of Nice)

**12:30-14:00: Lunch**

**Session 2:**

**14:00-15:00**
Quadratic optimal control of linear complementarity systems
Presented by Bernard Brogliato (INRIA, Rhône-Alpes)

**15:00-15:30: Break**

**15:30-16:30**
On the existence and uniqueness of anisotropic friction problems
Presented by Manuel Monteiro Marques (University of Lisbon)

**18:30-……: Dinner**

**Program September, 13th**

**08:30-09:00: Welcome, coffee**

**Session 3:**

**09:00-10:00**
Nonsmooth modal analysis overview
Presented by Mathias Legrand (McGill University)

**10:00-11:00**
Gears - how to simulate their elastic transient behavior considering many and frequently changing contacts
Presented by Peter Eberhard (University of Stuttgart)

**11:00-11:30: Break**

**Session 4:**

**11:30-12:30**
A boundary layer approach for mechanical systems with frictional contacts
Presented by Sotirios Natsiavas (Aristotle University, Thessaloniki)

**12:30-14:00: Lunch**

**14:00-15:00**
Transformations of chaotic attractors in piecewise smooth maps
Presented by Viktor Avrutin (University of Stuttgart)

**15:00-15:30: Break**

**15:30-16:30**
A self-similar topological structure in the dynamics of the one-dimensional bouncing ball system
Presented by Remco Leine (University of Stuttgart)

The abstracts of the talks are included below:

**Session 1:**

**Olivier Brüls**
**University of Liège, Belgium**

**Title: Formulation and discretization of non-penetration constraints and impact laws**

**Abstract:** This talk addresses modelling and simulation methods for rigid and flexible multibody systems with contact conditions. Following the nonsmooth contact dynamics approach, a contact condition is modelled as a unilateral constraint on the gap distance between the two bodies. The non-penetration condition should be supplemented with an impact law to obtain a consistent model. The equations of motion can then be obtained according to Moreau's sweeping process and discretized according to the Moreau-Jean scheme so that the impact law as enforced as a constraint on the system velocities between successive time steps. As a consequence, the non-penetration constraint is only approximately satisfied at position level. In this talk, we study alternative formulations of the unilateral constraints in the equations of motion along with their time discretization. We demonstrate the possibility to compute a numerical solution which simultaneously satisfies the unilateral (and bilateral) constraints at position-, velocity- and acceleration-levels. Several examples of 2D rigid and flexible multibody systems are presented to illustrate these properties.

**Malte Krack**
**University of Stuttgart, Germany**

**Title: How does a non-smooth absorber work under near-resonant forcing?**

**Abstract:** A linear vibration absorber can be tuned to effectively suppress the resonance of a particular vibration mode. It relies on the localization and dissipation of vibration energy in the absorber at a fixed and known frequency. Nonlinear energy sinks (NES) have a similar working principle. They are effective in a much wider frequency band but generally only in a limited amplitude range. To design NES, their working principle must be thoroughly understood. We consider a particular type of NES, a small mass undergoing impacts and dry friction within a cavity of a base structure (vibro-impact NES). The intriguing nonlinear dynamics under near-resonant forcing are illustrated. We then investigate how the energy transfer to higher vibration modes and the energy dissipation via inelastic impacts and dry friction contribute to the vibration suppression. Moreover, we assess the effectiveness of the vibro-impact NES for suppressing multiple resonances in comparison to a linear vibration absorber and a pure friction-damper with the same mass.

Stéphane Junca, Huong Le Thi, University of Nice, France
Mathias Legrand, Anders Thorin, McGill University, Canada

**Title: Dynamics near periodic grazing orbits for vibro-impact systems through the first return time to the contact surface**

**Abstract:** A theoretical study of the nonsmooth dynamics of a N-degree-of-freedom involving one unilateral constraint is addressed. The worst nonsmooth dynamics appears near orbits with a grazing contact. This is known since Arne Nordmark's results on "grazing bifurcations" and the corresponding "square root singularity" in 2001. Arne Nordmark uses a bifurcation analysis. We try to consider the whole dyamics near such orbits. For this purpose the simplest case is considered, the free dynamics is assumed to be linear. The study of such systems is motivated by the recent discovery of many nonlinear modes for a large number of degree of freedom. The first return time to the contact hyperplane can be set precisely which defines the whole set of initial data related with the nonsmooth dynamics. The first return time is nonsmooth only near initial data of grazing orbits. The singularity is usually a square root but can be a worst singularity with a smaller power which depends on the number of degree of freedom. Moreover, this first return time is discontinuous. Then, consequences due to the behavior of the first return time follow, the expected instability of grazing orbits, the behavior of the Poincaré map which contains all the nonlinear dynamics.

## Session 2:

Bernard Brogliato
INRIA, France

**Title: Quadratic optimal control of linear complementarity systems**

**Abstract:** Basing on the seminal results by Guo and Ye (SIAM J. Control Optim. 2016), we derive first-order necessary conditions for the quadratic optimal control of linear complementarity systems (LCS). Roughly speaking these conditions take the form of a boundary value problem where the state and the adjoint state obey an LCS with an inequality constraint. A direct method involving MPEC problems is also studied. The indirect method is solved with a code developed from a shooting method. Numerical results for both the direct and the indirect methods are shown on various types of LCS.

Manuel Monteiro Marques
University of Lisbon, Portugal

**Title: On the existence and uniqueness of anisotropic friction problems**

In this presentation, some preliminary results about existence and uniqueness of a class of anisotropic friction problems are discussed. The presentation focusses on the motion of a pointmass on a rough halfspace with (anisotropic) friction properties. In the isotropic case, the problem can be cast in a differential inclusion with a maximal monotone operator, for which existence and uniqueness has been shown, e.g. by Brezis. Standard descriptions of anisotropic friction also lead to the consideration of maximal monotone operators. However, a more sophisticated anisotropic friction law (as in Walker&Leine, ENOC, 2017) does not enjoy maximal monotonicity. We try to access the existence and uniqueness questions by putting the problem in a form similar to the sweeping process. A brief description of mathematical results for some classes of related non-convex or non-monotone problems – which may be useful in other settings – will be provided, if time allows.

**Session 3:**

Mathias Legrand
McGill University, Canada

**Title: Nonsmooth modal analysis overview**

**Abstract:** Nonsmooth modal analysis aims at computing modeshapes and attendant frequencies of vibration of structural systems subject to unilateral contact constraints. Nonsmooth modes of vibration are defined as one-parameter continuous families of nonsmooth periodic orbits satisfying the local equation together with the linear and nonlinear boundary conditions. A one-dimensional rod system is considered for illustration purposes with various boundary conditions one of which being unilateral. Semi-discretization in space via traditional Finite Element formulations is known to induce difficulties in the formulation, notably in the form of an impact law which generates chattering. Instead, the analysis is performed using the wave finite element method which is a shock-capturing finite volume method. The spectrum of vibration shown in the form of backbone curves provides valuable insight on the dynamics. In contrast to the linear system whose modes of vibration are standing harmonic waves, the nonsmooth modes of vibrations are traveling waves stemming from the unilateral contact condition. It is also shown that the vibratory resonances of the periodically driven system with light structural damping are well predicted by nonsmooth modal analysis. Furthermore, the initially unstressed and prestressed configurations exhibit stiffening and softening behaviors, respectively, as expected. Possible extensions in multi-dimensional frameworks are suggested.

Peter Eberhard, Pascal Ziegler and Lorin Kazaz
University of Stuttgart, Germany

**Title: Gears - how to simulate their elastic transient behavior considering many and frequently changing contacts**

**Abstract:** The simulation of elastic gear systems is highly complex for several reasons. While the precise contact line for rigid involute gears can even be obtained analytically, for more complex tooth shapes with corrections a sophisticated contact search has to be performed. Even worse, deformations change the tooth geometry slightly so that usually the contact is load dependent and distributed over several teeth. In this talk, we want to show how fully elastic transient contacts between many teeth can be computed. The obtained quality of results is close to reference finite element computations while the computation times are several orders of magnitude smaller. This makes it possible to compute the behavior over many full revolutions and for very complex tooth shapes such as for beveloid or hypoid gears.

## Session 4:

Sotirios Natsiavas
Aristotle University, Thessaloniki, Greece

**Title: A boundary layer approach for mechanical systems with frictional contacts**

**Abstract:** In the first part, this study focuses on the development of a new formulation, which describes the dynamics of a class of mechanical systems involving a single contact with friction. The whole process is performed within the general framework of analytical dynamics. At the same time, the efforts are assisted and enhanced by employing some fundamental tools of differential geometry. This provides a strong theoretical foundation, allowing for a systematic and consistent application of Newton's law of motion to systems possessing configuration manifolds with boundary. The manifold boundary is first defined by using the unilateral constraint representing the contact event. Then, it is shown that the corresponding contact phase takes place inside a thin layer near this boundary, where the dominant dynamics is described by a suitable set of three ordinary differential equations. The explicit appearance of time in the formulation introduces a short but finite time scale, which permits the consideration of contact of any combination of mechanical bodies, including rigid and deformable bodies. Next, the study is extended to cover cases of multiple contacts, by considering configuration manifolds with corners. Finally, the study concludes with presentation of a selected set of examples. In all cases, emphasis is put on illustrating the basic steps of the analysis as well as in capturing and investigating phenomena arising during central or eccentric collision of solid bodies.

Viktor Avrutin
University of Stuttgart, Germany

**Title: Transformations of chaotic attractors in piecewise smooth maps**

**Abstract:** What happens to a chaotic attractor when parameters are varied? For at least 20 years it is known that in piecewise smooth maps chaotic attractor may be persistent under parameter variation. It has also be shown that they may undergo several bifurcations, sometimes referred to as crises, associated with homoclinic bifurcations of some repelling cycles. Recently it was demonstrated that such bifurcations may form impressive bifurcation scenarios.

In my talk, I will focus on the most simple case, namely on chaotic attractors in 1D maps. I will compare some properties of chaotic attractors in three classes of maps: smooth, piecewise smooth continuous and piecewise smooth discontinuous. It appears that in the latter case multi-band chaotic attractors may be acyclic, while in the other cases they cannot.

I will give a brief overview of bifurcations a chaotic attractor may undergo, classifying them according to the eigenvalues of the corresponding repelling cycles. Being rather simple, this classification turns out to be of a great help for recognizing phenomena observed in systems of practical interest. Based on that I will present two bifurcation scenarios (bandcount adding and bandcount incrementing) formed by multi-band chaotic attractors in piecewise smooth maps. I will show how these scenarios are connected to the scenarios organizing the adjacent periodic domain (period adding and period incrementing, respectively).

To conclude, I will discuss an unexpected phenomenon (chaos far beyond numerical observability) recently discovered in a class of piecewise smooth maps originating from industrial applications from the field of power electronics.

Remco I. Leine, University of Stuttgart, Germany
Kilian Schindler, EPFL

**Title: A self-similar topological structure in the dynamics of the one-dimensional bouncing ball system**

**Abstract:** In this talk, paradoxical simulation results of the one-dimensional harmonically excited bouncing ball system are investigated. Chaos-like motion of the bouncing ball system with intermittent chattering (Zeno behaviour) is observed in simulations if the relative acceleration of the table exceeds a critical value. The motion irregularly visits an accumulation point (chattering) after which the ball is in contact with the table for a while and then lifts off again. However, one can show that a motion with repeated chattering is necessarily periodic and, therefore, that chaotic motion with chattering is theoretically impossible. A detailed analysis is given by looking at the backward and forward dynamics of grazing solutions. It is shown in detail that discontinuities in the impact map intersect infinitely many times and create a self-similar, very interweaved and extremely stretched topological structure if the

relative acceleration of the table exceeds the critical value. Small finite disturbances, for instance due to numerical simulation error, may deviate the respective trajectory from its theoretical evolution and make it dynamically switch between different regions of attraction, resulting in chaos-like behavior or, for instance, irregular motion with repeated chattering. The practical relevance of these findings consists in the fact that perturbations, caused by numerical errors in simulations, are equally present in the real physical system in form of model inaccuracies and actual unavoidable disturbances. The results are therefore strong indications pointing towards a fractalization-like dynamical mechanism interfering with the stability properties of several motions of the bouncing ball system.