A robot inspired by a non-smooth point mass model of a worm

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A non-smooth point mass model which mimics the locomotion of an earthworm is presented. The planar model consists of a chain with five elements on a rough ground. The distance between each pair of neighbouring elements is restricted by unilateral constraints. The contacts between the elements and the ground are subjected to anisotropic Coulomb friction. First, the equations of motion governing the impact-free motion are derived. Then, the impact equations are formulated which, together with the generalized Newton's impact law, describe the dynamics at instantaneous impacts.

1 Introduction

The locomotion of an earthworm is generated by waves of muscular contractions which alternately shorten and lengthen the body of the worm. The shortened part of the body is anchored to the ground by tiny claw-like bristles. The worm's locomotion can be roughly approximated by dividing the worm into identical segments which are either contracted or lengthened. Therefore, we have considered a planar model of a chain with five point masses on a rough ground as shown in Fig. 2. An actuator is placed between each pair of neighbouring elements in order to change the gap distance between them, which is limited by unilateral constraints. The effect of the worm's bristles are approximated by anisotropic Coulomb friction. The resulting non-smooth point mass model is able to mimic the locomotion of an earthworm in simulation. Based on the model, we have built the robot shown in Fig. 1.

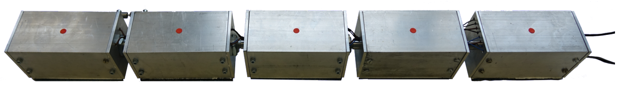

Fig. 1: Prototype of the worm robot consisting of rigid blocks which are actuated by four electromagnets.

2 Mechanical Model

For an introduction to non-smooth mechanics, we refer to [1]. The model consists of five elements (point masses) with mass m on a rough horizontal ground in the gravitational field g. The positions of the elements are represented by the generalized coordinates $\mathbf{q} = (q_1, q_2, q_3, q_4, q_5)^{\mathrm{T}}$. The velocities of the elements are given by the time derivative of the generalized coordinates, i.e. by $\dot{\mathbf{q}} = (\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5)^{\mathrm{T}}$. The pre- and post-impact velocities are denoted by $\dot{\mathbf{q}}^- = \left(q_1^-, q_2^-, q_3^-, q_4^-, q_5^-\right)^{\mathrm{T}}$ and $\dot{\mathbf{q}}^+ = \left(q_1^+, q_2^+, q_3^+, q_4^+, q_5^+\right)^{\mathrm{T}}$, respectively. These are needed for the description of velocity jumps due to instantaneous impacts. The latter occur, for example, if two elements collide with a non-zero relative velocity.

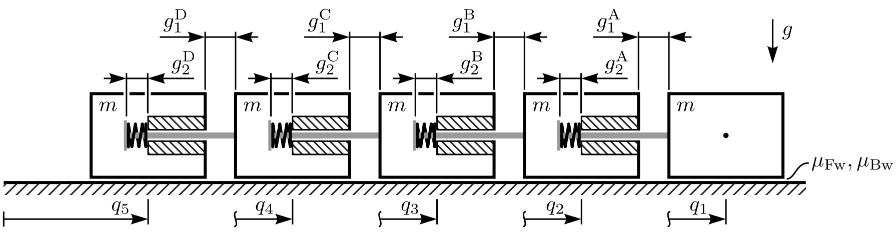

Fig. 2: Model of the worm robot consisting of five elements of mass m.

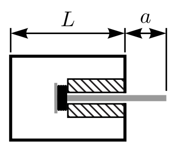

Fig. 3: Geometry of an element.

There are four actuators designated by A, B, C and D. Each of them is composed of an electromagnet together with a linear spring. The core of the electromagnet comes with a head which prevents it from being pulled out of the actuator. This limit stop and the fact that two neighbouring elements may not penetrate another are modelled by unilateral constraints. For each actuator I with $\mathrm{I} \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$, we introduce two gap functions g_1^{I} and g_2^{I}. The function g_1^{I} describes the gap distance

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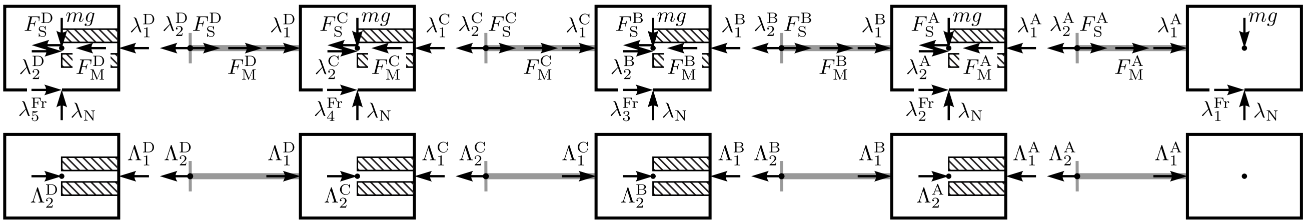

Fig. 4: Free-body diagram of the worm robot.

between the two neighbouring elements of the I-th actuator, while g_2^{I} measures the position of its core. The resulting eight gap functions can be gathered in a vector $\mathbf{g}$ as

$$\mathbf{g} = \left(g_1^{\mathrm{A}}\ g_2^{\mathrm{A}}\ g_1^{\mathrm{B}}\ g_2^{\mathrm{B}}\ g_1^{\mathrm{C}}\ g_2^{\mathrm{C}}\ g_1^{\mathrm{D}}\ g_2^{\mathrm{D}}\right)^{\mathrm{T}}, \quad \text{where} \quad \begin{pmatrix} g_1^{\mathrm{A}} \\ g_1^{\mathrm{B}} \\ g_1^{\mathrm{C}} \\ g_1^{\mathrm{D}} \end{pmatrix} = \begin{pmatrix} q_1 - q_2 - L \\ q_2 - q_3 - L \\ q_3 - q_4 - L \\ q_4 - q_5 - L \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} g_2^{\mathrm{A}} \\ g_2^{\mathrm{B}} \\ g_2^{\mathrm{C}} \\ g_2^{\mathrm{D}} \end{pmatrix} = \begin{pmatrix} q_2 - q_1 + L + a \\ q_3 - q_2 + L + a \\ q_4 - q_3 + L + a \\ q_5 - q_4 + L + a \end{pmatrix}. \quad (1)$$

The lengths L and a are introduced in Fig. 3. The contact velocities are given by $\boldsymbol{\gamma} := \dot{\mathbf{g}} = \left(\gamma_1^{\mathrm{A}}\ \gamma_2^{\mathrm{A}}\ \gamma_1^{\mathrm{B}}\ \gamma_2^{\mathrm{B}}\ \gamma_1^{\mathrm{C}}\ \gamma_2^{\mathrm{C}}\ \gamma_1^{\mathrm{D}}\ \gamma_2^{\mathrm{D}}\right)^{\mathrm{T}}$ from which we can deduce the generalized force directions $\mathbf{W}$ as

$$\boldsymbol{\gamma} = \dot{\mathbf{g}} = \frac{\partial \mathbf{g}}{\partial \mathbf{q}} \dot{\mathbf{q}} = \mathbf{W}^{\mathrm{T}} \dot{\mathbf{q}}, \quad \text{with} \quad \mathbf{W}^{\mathrm{T}} := \frac{\partial \mathbf{g}}{\partial \mathbf{q}}. \quad (2)$$

As before, the pre- and post-impact contact velocities are denoted by $\boldsymbol{\gamma}^- = \mathbf{W}^{\mathrm{T}}\dot{\mathbf{q}}^-$ and $\boldsymbol{\gamma}^+ = \mathbf{W}^{\mathrm{T}}\dot{\mathbf{q}}^+$, respectively.

A free-body diagram of the worm robot is shown in Fig. 4. The generalized contact forces which can be written in vector form as $\boldsymbol{\lambda} = \left(\lambda_1^{\mathrm{A}}, \lambda_2^{\mathrm{A}}, \lambda_1^{\mathrm{B}}, \lambda_2^{\mathrm{B}}, \lambda_1^{\mathrm{C}}, \lambda_2^{\mathrm{C}}, \lambda_1^{\mathrm{D}}, \lambda_2^{\mathrm{D}}\right)^{\mathrm{T}}$ obey the Signorini condition

$$0 \le \boldsymbol{\lambda} \perp \mathbf{g} \ge 0 \quad \Leftrightarrow \quad \lambda_j^I \ge 0,\ g_j^I \ge 0, \text{ and } \lambda_j^I g_j^I = 0, \quad (3)$$

for all $\mathrm{I} \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$ and $j \in \{1, 2\}$. The friction between element $k \in \{1, 2, 3, 4, 5\}$ and the ground is modelled as anisotropic Coulomb friction depending on the two friction coefficients μ_{Fw} and μ_{Bw}, s.t.

$$-\lambda_k^{\mathrm{Fr}} \in \frac{\mu_{\mathrm{Fw}} + \mu_{\mathrm{Bw}}}{2} \lambda_N \mathrm{Sgn}(\dot{q}_k) + \frac{\mu_{\mathrm{Fw}} - \mu_{\mathrm{Bw}}}{2} \lambda_N, \quad \text{with} \quad \lambda_N = mg, \quad \text{and} \quad \mathrm{Sgn}(v) := \begin{cases} -1 \text{ if } v < 0, \\ [-1, 1] \text{ if } v = 0, \\ 1 \text{ if } v > 0. \end{cases} \quad (4)$$

The friction forces can be written in vector notation as $\boldsymbol{\lambda}^{\mathrm{Fr}} = \left(\lambda_1^{\mathrm{Fr}}, \lambda_2^{\mathrm{Fr}}, \lambda_3^{\mathrm{Fr}}, \lambda_4^{\mathrm{Fr}}, \lambda_5^{\mathrm{Fr}}\right)^{\mathrm{T}}$. The electromagnetic force of actuator $\mathrm{I} \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$ is modelled as $F_{\mathrm{M}}^{\mathrm{I}} = f_{\mathrm{I}}\left(g_2^{\mathrm{I}}(\mathbf{q}), t\right)$ where f_{I} is defined by the characteristic curve of the electromagnet and by its switching sequence. The forces exerted by the linear springs are given by $F_{\mathrm{S}}^{\mathrm{I}} = k_{\mathrm{I}}(g_2^{\mathrm{I}}(\mathbf{q}) - l_0)$ where k_{I} denotes the spring stiffness, l_0 its unstressed length and $\mathrm{I} \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$. The impact-free dynamics is then given by

$$\mathbf{M}\ddot{\mathbf{q}} - \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{W}_{\mathrm{Fr}}\, \boldsymbol{\lambda}^{\mathrm{Fr}} + \mathbf{W}\boldsymbol{\lambda} \text{ with } \mathbf{M} = m\mathbf{I}_{5,5},\ \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, t) = \begin{pmatrix} F_{\mathrm{S}}^{\mathrm{A}}(\mathbf{q}) + F_{\mathrm{M}}^{\mathrm{A}}(\mathbf{q}, t) \\ -F_{\mathrm{S}}^{\mathrm{A}}(\mathbf{q}) - F_{\mathrm{M}}^{\mathrm{A}}(\mathbf{q}, t) + F_{\mathrm{S}}^{\mathrm{B}}(\mathbf{q}) + F_{\mathrm{M}}^{\mathrm{B}}(\mathbf{q}, t) \\ -F_{\mathrm{S}}^{\mathrm{B}}(\mathbf{q}) - F_{\mathrm{M}}^{\mathrm{B}}(\mathbf{q}, t) + F_{\mathrm{S}}^{\mathrm{C}}(\mathbf{q}) + F_{\mathrm{M}}^{\mathrm{C}}(\mathbf{q}, t) \\ -F_{\mathrm{S}}^{\mathrm{C}}(\mathbf{q}) - F_{\mathrm{M}}^{\mathrm{C}}(\mathbf{q}, t) + F_{\mathrm{S}}^{\mathrm{D}}(\mathbf{q}) + F_{\mathrm{M}}^{\mathrm{D}}(\mathbf{q}, t) \\ -F_{\mathrm{S}}^{\mathrm{D}}(\mathbf{q}) - F_{\mathrm{M}}^{\mathrm{D}}(\mathbf{q}, t) \end{pmatrix}, \quad (5)$$

where $\mathbf{I}_{5,5}$ denotes a five-by-five identity matrix, $\mathbf{W}_{\mathrm{Fr}} = \mathbf{I}_{5,5}$ and $\mathbf{W}$ is given by (2). The impact dynamics is described by the impact equations together with the generalized Newton's impact law [2] with restitution coefficient $\varepsilon \in [0, 1]$, i.e.

$$\mathbf{M}(\dot{\mathbf{q}}^+ - \dot{\mathbf{q}}^-) = \mathbf{W}\boldsymbol{\Lambda} \quad \text{and} \quad 0 \le \boldsymbol{\Lambda} \perp (\boldsymbol{\gamma}^+ + \varepsilon\boldsymbol{\gamma}^-) \ge 0, \quad (6)$$

with $\boldsymbol{\Lambda} = \left(\Lambda_1^{\mathrm{A}}, \Lambda_2^{\mathrm{A}}, \Lambda_1^{\mathrm{B}}, \Lambda_2^{\mathrm{B}}, \Lambda_1^{\mathrm{C}}, \Lambda_2^{\mathrm{C}}, \Lambda_1^{\mathrm{D}}, \Lambda_2^{\mathrm{D}}\right)^{\mathrm{T}}$. We have implemented the model for numerical simulation using Moreau time-stepping [1].

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