A SET-VALUED FORCE LAW FOR SPATIAL COULOMB-CONTENSOU FRICTION

Remco I. Leine   Christoph Glocker

Institute of Mechanical Systems,
Center of Mechanics, ETH Zentrum,
CH-8092 Zürich, Switzerland,
remco.leine@imes.mavt.ethz.ch
christoph.glocker@imes.mavt.ethz.ch

The aim of this paper is to develop a contact law for combined spatial Coulomb friction and normal friction torque (drilling friction) as a function of sliding velocity and spin. We will call this extended contact law the Coulomb-Contensou friction law. The Contensou phenomenon occurs for instance in an electric polishing machine with turning brushes used to clean floors. The machine is hard to move when the brushes are non-rotating (Coulomb friction) but the machine can easily be pushed over the floor with rotating brushes (Contensou phenomenon).
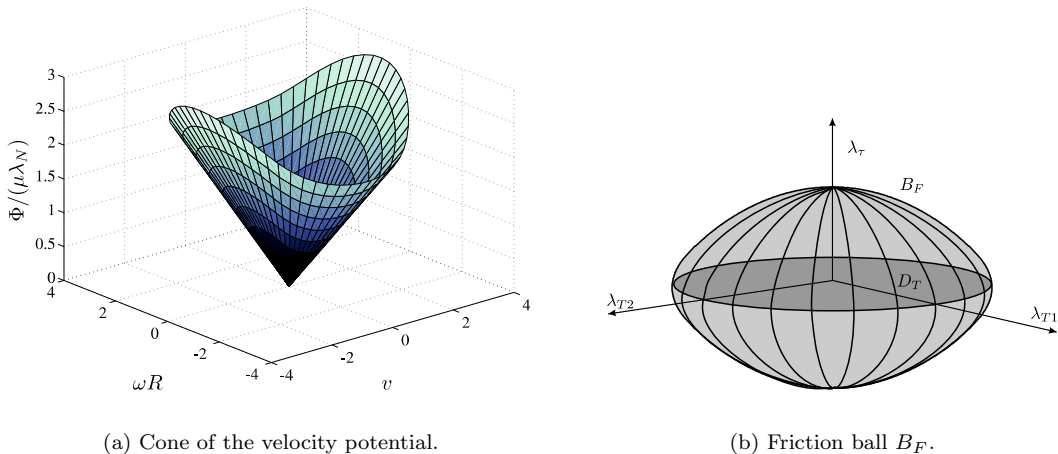

(a) Cone of the velocity potential.   (b) Friction ball B_F.

Figure 1: The velocity potential Φ and set of admissible contact forces and torque B_F.

The Coulomb-Contensou friction law shows a continuous behaviour for non-zero sliding velocity and spin and a set-valued behaviour for zero sliding velocity and spin (stick). Coulomb's friction law is generally formulated as a set-valued force law within a framework of convex analysis [1, 3]. The formulation of frictional contact problems of mechanical rigid multi-body systems with set-valued contact laws leads to a nonlinear algebraic inclusion, which is complicated to solve. The first (and often worst) method is to regularize the set-valued force laws and obtain a stiff problem with an inexact solution. Instead, one simply has to accept the set-valued nature of the problem and deal with the nonlinear algebraic inclusion. The present study formulates the contact problem for spatial Coulomb-Contensou friction in the framework of potential theory and convex analysis [2]. The velocity potential Φ for Coulomb-Contensou friction has been derived as function of the sliding

speed v and the angular rotation speed ω and is shown in Figure 1a. The set-valued force law can be derived from the non-smooth velocity potential Φ. The set of admissible friction forces and normal friction torque is depicted in Figure 1b.

The theory and numerical methods are applied to the Tippe-Top, a toy top which inverts its orientation when spun fastly (see Figure 2). The gyroscopical moment which inverts the orientation of the top is friction induced. The analysis and numerical results on the Tippe-Top illustrate the importance of Coulomb-Contensou friction for the dynamics of systems with friction.

(a) The top in steel.
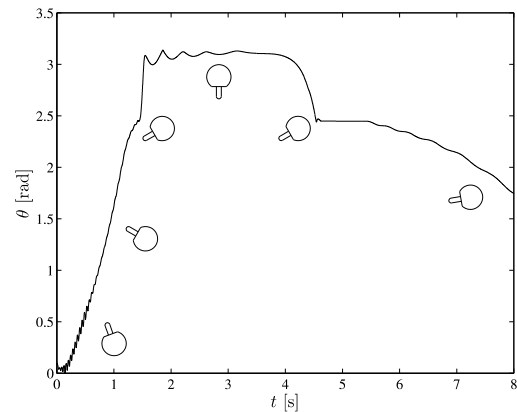
(b) Inclination

Figure 2: The Tippe-Top

References

[1] Ch. Glocker. *Set-Valued Force Laws, Dynamics of Non-Smooth Systems*, volume 1 of *Lecture Notes in Applied Mechanics*. Springer-Verlag, Berlin, 2001.

[2] R. I. Leine and Ch. Glocker. A set-valued force law for spatial Coulomb–Contensou friction. *European Journal of Mechanics – A/Solids*, 22:193–216, 2003.

[3] R. I. Leine, Ch. Glocker, and D. H. Van Campen. Nonlinear dynamics of the woodpecker toy. *Proceedings of DETC 2001 ASME Symposium on Nonlinear Dynamics and Control In Engineering Systems*, September 9-12 2001. Pittsburgh, CD-ROM, DETC2001/VIB-21608.