Experimental and numerical nonlinear modal analysis of a beam with impact: Part II - Experimental investigation

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Summary Methods for nonlinear experimental modal analysis, such as nonlinear phase resonance testing, are currently limited to smooth nonlinearities. However, the modeling of systems with contact interactions, being relevant in many industrial applications, often leads to models with strong nonsmooth nonlinearities. This contribution investigates the capability of nonlinear phase resonance testing to analyze systems with nonsmooth nonlinearities. Therefore, a series of automated steady-state measurements with phase controlled excitation is used. The experimental investigation is complemented with the numerical results obtained in Part I of this study.

Keywords nonlinear modes, nonlinear phase resonance testing, contact interactions, nonsmooth systems, nonlinear system identification

Intoduction

In many industrial applications contact interactions significantly influence the dynamic behavior, leading to models with nonsmooth nonlinearities. However, the identification and modeling of these nonsmooth nonlinearities, such as impacts and unilateral constraints, is still a difficult endeavor. For instance, impacts lead to local impulsive forces which are difficult to measure directly in the contact interface and require high experimental effort due to the short duration of the contact phase.

Therefore, this contribution proposes a method to capture the global behavior of structures subjected to impacts in the framework of nonlinear modes. The concept of nonlinear modes is widely applied for the theoretical and numerical analysis of nonlinear structures, while its application to the experimental identification of nonlinear structures is still in its infancy. The commonly used manual force appropriation lacks robustness for strong nonlinearities and the timefrequency analysis of the free-decay, with which it is often combined, limits the resolution of the extracted backbone curves. Therefore, this method seems to be infeasible for the experimental extraction of amplitude dependent nonlinear modal characteristics of nonsmooth systems. A recent method for nonlinear experimental modal analysis [2], which uses a series of steady-state measurements with phase controlled excitation provides a more robust experimental procedure and can be exploited for the analysis of nonsmooth systems. The robustness and accuracy of this method for systems with strong nonsmooth nonlinearities is investigated based on a benchmark beam structure with impact and compared to numerical simulations performed with the method proposed in Part I of this study [3].

Experimental Procedure and Benchmark Structure

The method for the experimental extraction of nonlinear modes is based on the phase resonance method proposed by Peeters [1]. Thus, the method aims at extracting periodic motions which can be described by the conservative nonlinear equation of motion

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) + \mathbf{f}^{\mathrm{nl}}(\mathbf{q}(t)) = \mathbf{0}.$$
(1)

In contrast, the real structure is damped and has to be excited to enforce a periodic motion. In theory, this can be accomplished by a spatially distributed force vector which is shifted in phase by 90° with respect to all harmonics of the displacement for all points of the structure. In practice, for weakly damped systems with smooth nonlinearities, often a single point single harmonic excitation force with the aforementioned phase lag gives satisfactory results, i.e., is capable of isolating the nonlinear mode. This simple approximation is retained for the case of severe nonsmooth nonlinearities and it is shown for the benchmark example that the nonlinear modes can be isolated in this case with a reasonable accuracy. To control the phase of the fundamental harmonic force a phase-locked loop (PLL) controller



Figure 1: Photo and schematic sketch of the experimental setup.

as proposed in [2] is used. The PLL generates a harmonic voltage signal with a frequency which is controlled such that the phase lag between the fundamental harmonic of the force and the displacement is 90°. The PLL provides a simple control concept with a high robustness in the case of strong nonlinear distortions, that are to be expected due to nonsmooth nonlinearities.

The benchmark structure used in this study consists of a steel beam subjected to a rigid stop at one end. The stop is realized by a steel contact element mounted on a load cell to measure the impact force. A photo of the test rig is shown in Fig. 1. The structure is excited by an electrodynamic shaker in the first bending mode, the excitation force is measured with a load cell and the response with six accelerometers. The excitation level is increased incrementally from a low level, where the gap between stop and beam remains open, i.e., the structure behaves linear, up to a high level where the beam is in contact with the impact element. For the PLL controller the excitation force and the displacement at the excitation position are used as reference signals. For the measurement of the displacement a laser doppler vibrometer (LDV) with a displacement decoder is applied. The use of the displacement signal as reference provides, compared to the acceleration, the advantage that the displacement is less distorted by higher harmonics. Therefore, it is found that the control based on the displacement signal provides a higher robustness in the strongly nonlinear regime. A schematic sketch in Fig. 1 summarizes the experimental procedure.

Experimental Results

The PLL based nonlinear modal analysis method as described above is applied to the test structure shown in Fig. 1. Furthermore, a numerical model of the test structure which consists of 21 beam elements subjected to a hard contact at the tip is used to calculate the nonlinear modes numerically with the method described in Part I (see [3]). In contrast to the experimental results, for which an approximate force appropriation is used, the numerical results are obtained by directly solving Eq. (1) describing the motion of the conservative system. Exemplary, the experimental and numerical results for the backbone curve for the point at the beam tip, i.e., the contact point, are shown in Fig. 2. It can be seen that in the measured amplitude range both curves agree very well for a numerical calculation with five harmonics as described in detail in Part I [3]. There is a small difference of both backbone curves suggesting the contact in the experiment is slightly softer compared to the numerical calculation. This difference may be caused by the fact that in the numerical simulation the contact is regarded as perfectly rigid, whereas in reality the contact element has some compliance. However, it is emphasized that the numerical results are obtained without the need for the identification of any impact parameters or any calibration of the nonlinear numerical model. Solely, the linear parameters of the beam have been identified to match the linear eigenfrequencies. The agreement of the simulations and the measurements also indicate that the isolation of the nonlinear mode is possible to a satisfying accuracy even with the rough approximation of a single harmonic single point force controlled with the PLL. A more detailed comparison of the displacement at the beam tip shows an almost perfect agreement of simulation and experiment. However, there is some deviation in the contact force. Firstly, the maximum contact force in the simulation significantly exceeds the contact force in the experiment. Secondly, the contact duration is longer in the experiment and a small second peak can be seen. This peak suggests that there is a local high frequency vibration in the contact area.

This local high frequency vibration can also be observed in the velocity and acceleration signals measured at the beam tip. The velocity and acceleration signals illustrate the significantly nonlinear behavior of the beam. Particularly,



Figure 2: Left: Experimental and numerical backbone curve for the beam tip. Right: Comparison of contact force and displacement (top) and measured velocity and acceleration (bottom) for point P1.

the acceleration signal reveals strong high frequency content caused by the impact. The impulsive character of the very hard steel-to-steel contact manifests itself in the acceleration signal as a high peak of almost 1000 m/s^2 at the contact instant. Furthermore, a sudden jump in velocity can be observed due to the impact.

Conclusion

This contribution shows that recent steady state measurement methods can open the door towards the experimental analysis of nonlinear modes of nonsmooth systems. The experimental approach relies on a PLL controlled excitation force. The PLL controller proves to be robust even in the case of a severe impact nonlinearity. For the studied benchmark test the isolated nonlinear modes are in good agreement with the theoretical nonlinear modes calculated with the method proposed in Part I of this study. It is noted that this is the case even though only a single point force with phase control of the fundamental harmonic is applied, which can be seen as a rather rough approximation. The simplicity and robustness make the method appealing for the analysis of the complicated dynamics of impacting structures. Future work aims at investigating the generality of the proposed method for different test cases.

References

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