Experimental nonlinear modal analysis using a Phase-Locked-Loop

S. Peter[1] **and R. I. Leine**[1]

[1]Institute for Nonlinear Mechanics, University of Stuttgart
Stuttgart, Germany
peter@inm.uni-stuttgart.de, leine@inm.uni-stuttgart.de

Abstract In this contribution an efficient control-based method for phase resonance testing of nonlinear systems is proposed. A Phase-Locked-Loop (PLL) is used to maintain the phase lag quadrature criterion during the nonlinear modal test. Furthermore, a new type of nonlinear mode indicator function based on the excitation power is presented. The functionality of the method is illustrated by experimental results obtained for a benchmark beam structure.

One method for the experimental extraction of nonlinear normal modes (NNMs) is the phase resonance method proposed by Peeters [1]. With this method a NNM of the underlying nonlinear conservative system of a weakly damped real structure can be measured. The NNM motion is governed by the differential equation

$$\mathbf{M}\,\ddot{\mathbf{x}}(t) + \mathbf{K}\,\mathbf{x}(t) + \mathbf{f}^{\mathrm{nl}}(\mathbf{x}(t)) = \mathbf{0}. \tag{1}$$

For the isolation of the NNM an appropriate force must be applied to the structure to compensate for the damping without having any influence on the motion described by Eq. (1). Theoretically this can be accomplished by a spatially distributed force vector with a phase of 90° with respect to the displacement for the fundamental frequency and its higher harmonics. In practice, it is often sufficient to apply a single harmonic, single point force to approximately isolate the NNM.

Although this method proved its usefulness some practical issues arise. The manual tuning of the phase is a time consuming procedure. Furthermore, the use of the free decay for the extraction of the backbone curve can induce transient effects and requires sophisticated signal processing. To

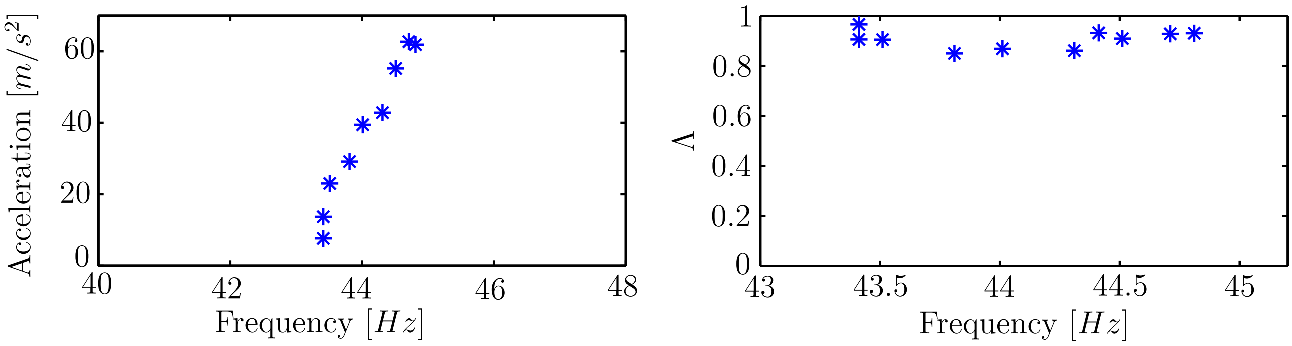

Figure 1: Left: Backbone curve for a reference node, Right: PBMIF value for the measurements.

circumvent these issues, a PLL controller can be used, which is capable of tuning the phase between two signals. For the NNM measurement, these signals can, for instance, be the displacement and the excitation force for which the phase difference is set to 90°. If the excitation amplitude is incrementally increased and the NNM frequency changes due to nonlinearity, then the excitation frequency is tuned by the PLL such that the phase lag quadrature is always maintained for the first harmonic. With this method a series of steady state measurements can be used to extract the backbone curve. Experimental results for the backbone curve of a beam with cubic nonlinearity are shown in Fig. 1. It can be observed that the PLL is capable of tracking the backbone curve also in the nonlinear range and clearly reveals the stiffening nonlinearity of the system.

For the analysis of the quality of the NNM isolation a new mode indicator function is used. This mode indicator function uses the relation between the active and apparent power of the excitation. The active power of a nonlinear system can be defined in analogy to Budeanu's definition in electrical engineering [2] as

$$P = \sum_{n=1}^{\infty} \mathbf{F}_{n,\mathrm{exc}}^{\mathrm{T}} \dot{\mathbf{X}}_n \cos(\varphi_n), \tag{2}$$

where $\mathbf{F}_{n,\mathrm{exc}}$ and $\dot{\mathbf{X}}_n$ are vectors of size $j \times 1$ with the effective values of the n-th harmonic of the force and velocity at j excitation points and φ_n is the phase angle of the n-th harmonic. The apparent power is defined using vectors with the root mean square values of these quantities

$$S = \mathbf{F}_{\mathrm{RMS}}^{\mathrm{T}} \dot{\mathbf{X}}_{\mathrm{RMS}}, \tag{3}$$

where each entry of the vectors $\mathbf{F}_{\mathrm{RMS}}$ and $\dot{\mathbf{X}}_{\mathrm{RMS}}$ corresponds to one excitation point k and can be calculated as $x_{k,\mathrm{RMS}} = \sqrt{\sum_{n=1}^{\infty} x_{nk}^2}$. The power based nonlinear mode indicator function (PBMIF) can be defined by relating the active to the apparent power

$$\Lambda := \frac{|P|}{S} \in [0,1]\,. \tag{4}$$

A value of unity indicates perfect NNM isolation, as this means that the velocity, i.e. the damping forces, for each harmonic are exactly out of phase and equal in amplitude to the excitation forces. The experimental results for the PBMIF shown in Fig. 1 indicate a good NNM isolation with the PLL method. The PBMIF concept can be regarded as an extension of the reactive power method [3], which uses the concept of active and reactive power but is limited to linear modal analysis. In nonlinear modal analysis, however, it is essential to consider the apparent power, as the power triangular relation known from linear systems theory does generally not hold for non-sinusoidal signals. The PBMIF provides a very general and reliable indication of the NNM isolation which is easy to implement experimentally as only measurements of the forces and velocities at the excitation points are required.

References

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