Experimental Frequency Response Synthesis for Nonlinear Systems

Simon Peter*, Maren Scheel**, Malte Krack** and Remco I. Leine *
* *Institute for Nonlinear Mechanics, University of Stuttgart, Germany*
***Institute of Aircraft Propulsion Systems, University of Stuttgart, Germany*

Summary. This contribution proposes a method for the synthesis of nonlinear frequency response functions based on the results of a linear and nonlinear experimental modal analysis. For the nonlinear modal analysis an automated phase resonance approach with a phase-locked loop controller is used, whereas the linear modes can be obtained with standard equipment. The nonlinear frequency response of the structure is directly synthesized with the results of these measurements, taking into account one nonlinear mode and all linear modes of the structure. The method is demonstrated experimentally on a benchmark beam structure and the results are validated against the frequency response measured with a commercial controller and a custom-made phase controller.

Introduction

Nonlinear frequency response functions (FRFs) are a widely used concept for the experimental identification and numerical analysis of nonlinear systems. Whereas the numerical calculation of nonlinear FRFs is nowadays commonly carried out even for industrial scale systems, their measurement still presents a number of difficulties. Particularly for weakly damped, strongly nonlinear structures, sharp peaks at resonance and jump phenomena in the frequency response complicate the measurements. Typically, sine-sweep measurements are conducted for different forcing levels which is a time consuming task. Therefore, this paper proposes a novel approach to synthesize the frequency response on the basis of experimentally obtained linear and nonlinear modal data. The measurement of linear modes has been a standard procedure for many years. Recent progress also allows for the reliable measurement of nonlinear modes with limited experimental effort. The obtained linear and nonlinear modal data can be used for the synthesis of a whole family of FRFs for different forcing and damping conditions without the need for any additional experimental effort or physical model of the structure. Thereby, the elaborate measurement of nonlinear FRFs can be replaced by two less demanding experimental techniques.

Experimental Method

The conservative nonlinear modes of a structure being subjected to weak damping can be measured with the phase resonance method. Hereto, an appropriate excitation force has to be applied that cancels out the damping forces without having an influence on the motion of the underlying conservative system. As a first approximation of this appropriate force, a single harmonic single point force can be used with a phase shift of 90° with respect to the displacement [1]. This excitation force can be generated using a phase-locked-loop (PLL) controller. The PLL controller allows for the approximate extraction of the nonlinear modal parameters of a branch of nonlinear modes [2]. A nonlinear mode j represents a periodic motion and can be described by a Fourier series as

$$\boldsymbol{x}_{\mathrm{nl,j}}(t) = \mathrm{Re}\left\{ \sum_{n=1}^{\infty} q_j \tilde{\boldsymbol{\phi}}_{n,j} e^{in\tilde{\omega}_{0,j}t} \right\}, \tag{1}$$

where $\tilde{\boldsymbol{\phi}}$ represents some normalized mode shape, q_j a modal amplitude and $\tilde{\omega}_{0,j}$ the nonlinear modal frequency. In contrast to linear modes, the nonlinear modal parameters depend on the amplitude, indicated by the $\tilde{()}$ symbol.
Once the linear and nonlinear modal parameters of the structure are known, it is supposed that the response of the system in the vicinity of a nonlinear mode can be calculated by applying the single nonlinear resonant mode method [3] yielding

$$\boldsymbol{x}(t) \approx \boldsymbol{x}_{\mathrm{nl,j}}(t) + \sum_{k\neq j}^{N_l} \boldsymbol{x}_{\mathrm{ln,k}}(t), \tag{2}$$

which can be regarded as a superposition of the nonlinear modal response and the contribution of all the linear modes of the structure. Of course, the superposition principle does not generally hold for nonlinear systems. However, in the case of well isolated modes that do not undergo excessive modal coupling, e.g. through internal resonances, it is reasonable to assume that most of the energy is confined to a single nonlinear mode j. If this is the case, then the contributions of the other modes are comparably small and can be described by a linearized model. For this reason, the nonlinear contribution to the frequency response can be calculated for a single harmonic excitation force by projection of the nonlinear mode shape onto the equation of motion of the forced and damped system. For the fundamental harmonic response this yields

$$[-\Omega^2 + 2i\Omega\tilde{\omega}_{0,j}\tilde{\delta}_j + \tilde{\omega}_{0,j}^2]q_j = \mathrm{Re}\left\{ \tilde{\boldsymbol{\phi}}_{1,j}^{\mathrm{T}} \boldsymbol{f}_{1,\mathrm{exc}} \right\}, \tag{3}$$

where Ω denotes the forcing frequency. Herein, the nonlinear modal parameters are amplitude dependent, such that Eq. (3) represents an implicit nonlinear equation which has to be solved iteratively for the modal amplitude q_j. The linear modal contributions can be calculated as it is known from linear theory and the total response is calculated according

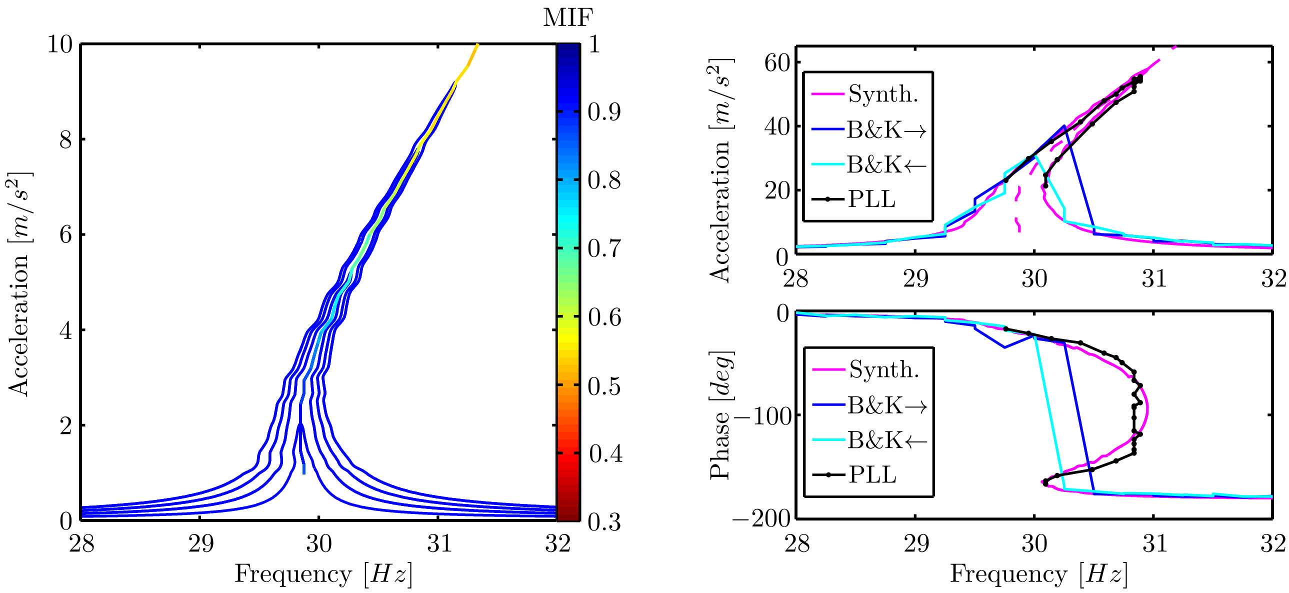

Figure 1: Left: Backbone curve with nonlinear mode indicator function (MIF) and synthesized frequency response for the excitation point for different forcing amplitudes. Right: Comparison of synthesized frequency response and phase with frequency response and phase for the beam tip measured with commercial controller (B&K, sweep up and down) and a custom-made phase controller (PLL)

to Eq. (2). A similar methodology has previously been used in numerical calculations for model reduction purpose [4]. In contrast, the method presented here does not require any numerical model for the synthesis but completely relies on experimental data.

Experimental Results and Validation

The synthesis method is applied to a benchmark structure consisting of a steel beam with a thin beam at the tip and clamped-clamped boundary conditions [2]. Due to the large deflection the thin beam causes a geometric nonlinearity, such that the system shows a Duffing-type stiffening behavior. The FRF is synthesized for this setup based on the linear and nonlinear experimental modal data. Fig. 1 (left) shows the measured backbone curve of the system with the color indicating the isolation quality of the nonlinear mode evaluated with a power based nonlinear mode indicator function [2]. The blue curves represent the synthesized frequency response for a sinusoidal excitation with different amplitudes. The synthesized responses are validated against the FRF data measured with a commercial controller and a custom-made phase controller (see Fig. 1 (right)). It can be seen that the commercial controller fails to capture the jump in the FRF correctly and the maximum amplitude is underestimated. The custom-made phase controller is capable of capturing the FRF around resonance correctly including the unstable branch. The FRF predicted with the proposed synthesis method agrees very well with the measured FRF obtained with the phase controller.

Conclusion

A novel approach for predicting the frequency response of a nonlinear system on the basis of experimental linear and nonlinear modal data is presented. Thereby, the difficulties related to nonlinear FRF measurements can be circumvented. The experimental synthesis is successfully applied to a benchmark beam structure with Duffing-type nonlinearity and the synthesized frequency response shows a good agreement with the measured frequency response. The possibility of calculating the response for different forcing and damping conditions without additional experimental effort offers a useful tool for system identification and model validation purposes. Future work involves the extension of the concept to nonlinearly damped structures and its application to more complex test cases.

References

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