Towards Experimental Nonlinear Modal Analysis of Systems with Nonlinear Damping

Maren Scheel*, Simon Peter**, Remco I. Leine** and Malte Krack*
* *Institute of Aircraft Propulsion Systems, University of Stuttgart, Germany*
** *Institute for Nonlinear Mechanics, University of Stuttgart, Germany*

Summary. A new method is proposed for experimentally extracting frequency, damping ratio and deflection shape of nonlinear modes as a function of the vibration level. The applicability to systems with nonlinear damping is possible by basing the approach on the extended periodic motion concept. Using feedback-control, the appropriated forcing required to isolate the nonlinear modes is experimentally realized. The method is numerically demonstrated for a model of an experimental setup involving a friction-damped beam, a shaker and a controller. The modal characteristics agree well with those obtained by numerical nonlinear modal analysis of the autonomous beam.

Introduction

In nonlinear structural dynamics, experiments are required for model validation and identification of properties that cannot be reliably determined by simulation. Nonlinear modes are a useful concept for extracting the vibration signature of nonlinear mechanical systems [1]. Therefore, modal characteristics are regarded as suitable metrics for nonlinear experimental characterization. Nonlinear modal testing procedures have been developed which extend the well-known phase resonance [2] and the phase separation [3] techniques. However, these methods are designed for the modal analysis of the underlying conservative system in the presence of weak damping, and cannot be used for identification of (nonlinear) damping caused, e.g., by friction joints. On the other hand, nonlinear damping identification methods are commonly based on vibration decay curves obtained using ring-down measurements [4]. In practice, the accuracy of these methods suffers from imperfect excitation removal and the limited number of sampling points on the decay curve in the presence of moderate and high damping.

To overcome the difficulties associated with the transient nature of damped nonlinear modes, the present method is based on steady-state vibrations. To this end, the nonlinear modes are defined in accordance with the extended periodic motion concept [5] described in the following. Consider an autonomous dynamical system governed by

$$\mathbf{M}\ddot{\boldsymbol{x}} + \mathbf{K}\boldsymbol{x} + \tilde{\boldsymbol{g}}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \mathbf{0}, \tag{1}$$

with the vector of generalized coordinates $\boldsymbol{x}$ measured from an equilibrium point $\boldsymbol{x} = \mathbf{0}$, mass and stiffness matrices $\mathbf{M} = \mathbf{M}^{\mathrm{T}} > 0$ and $\mathbf{K} = \mathbf{K}^{\mathrm{T}} > 0$. The vector $\tilde{\boldsymbol{g}}(\boldsymbol{x}, \dot{\boldsymbol{x}})$ contains linear and sufficiently smooth nonlinear restoring and damping forces. According to the conventional concept, a nonlinear mode is the nonlinear extension of an associated mode of the linearized system and occupies a two-dimensional invariant manifold in the system's phase space. In the presence of damping, the motions on this manifold typically decay. According to the extended periodic motion concept, the nonlinear modes are periodic motions of the autonomous surrogate system,

$$\mathbf{M}\ddot{\boldsymbol{x}} + \mathbf{K}\boldsymbol{x} + \tilde{\boldsymbol{g}}(\boldsymbol{x}, \dot{\boldsymbol{x}}) - \xi\mathbf{M}\dot{\boldsymbol{x}} = \mathbf{0}. \tag{2}$$

The motions are periodic when the artificial negative damping term $-\xi\mathbf{M}\dot{\boldsymbol{x}}$ is large enough to compensate the natural dissipation. The approach is exact in the conservative nonlinear case, where $\xi = 0$, and in the linear case with modal damping, where the modes are orthogonal with respect to the mass proportional term. However, the term may cause modal distortion in the presence of high damping and, at the same time, strong modal interactions. The nonlinear modes following this definition describe better the nonlinear steady-state dynamics near forced resonances and limit cycles induced by negative damping of a particular mode [5].

Experimental Approach

The term $-\xi\mathbf{M}\dot{\boldsymbol{x}}$ in Eq. (2) is an appropriated self-excitation applied to every material point. Although this can be easily included in simulation, it is impossible to realize experimentally. It is therefore investigated if this excitation can be approximated by a finite number of excitation points. Moreover, the direct velocity feedback is replaced by a phase resonant harmonic excitation. Hence, the realized excitation deviates from the theoretical one with regard to spatial distribution and higher frequency content. This can lead to an imperfect isolation of the nonlinear mode. The isolation quality can be estimated based on power quantities analogously to [6].

The proposed experimental approach is a two-step procedure. In the first step, a conventional linear experimental modal analysis is carried out for a low excitation level to extract the natural frequencies, damping ratios and mass-normalized deflection shapes of the underlying linear system. In the second step, the nonlinear modal testing is carried out using the phase resonant excitation for incrementally increasing load levels. This can be achieved with a phase-locked-loop (PLL) controller, which is known for its robustness and efficiency [6]. Thus, the backbone curve of the frequency response is tracked. The modal frequency and deflection shape can be directly monitored as a function of the vibration level. The nonlinear modal damping is estimated by power considerations. If the nonlinear mode is isolated perfectly, the active

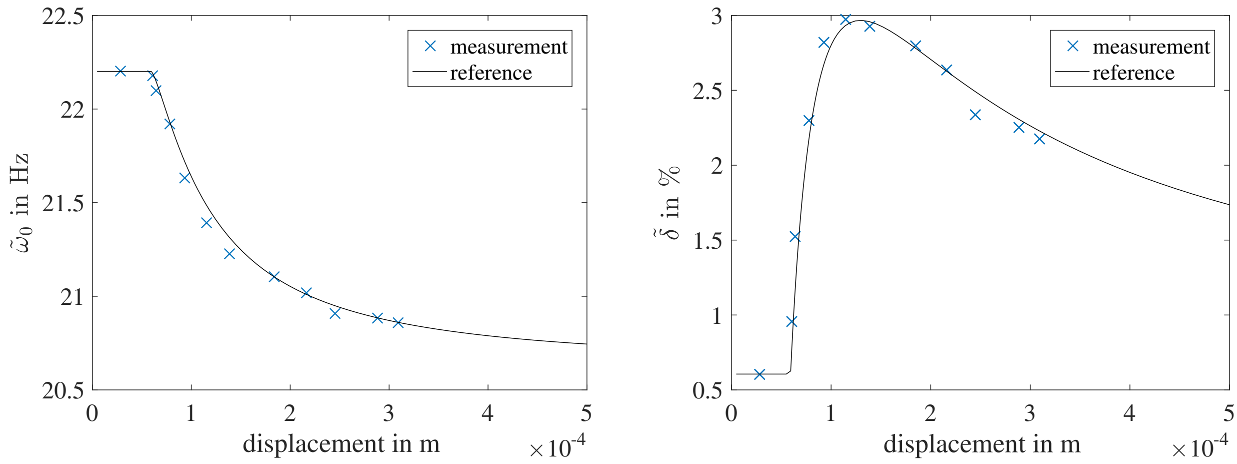

Figure 1: Natural frequency (left) and the modal damping ratio (right), as a function of the vibration level.

power of the excitation is equal to the dissipated active power,

$$P_{\mathrm{exc}} = \sum_k \frac{1}{2} f_{k,1} v_{k,1} \cos(\varphi_k) = \tilde{\delta}(q)\, \tilde{\omega}_0^3(q)\, q^2 = P_{\mathrm{diss}}. \tag{3}$$

Here, f_k, v_k and φ_k are the magnitudes of the fundamental harmonics of excitation force and velocity and the phase angle between force and velocity, for each excitation point k. q is the modal amplitude.

Numerical Validation

A pre-test simulation is carried out to numerically validate the proposed method. As specimen, a clamped-free beam is used where a friction nonlinearity is introduced in the form of an elastic Coulomb element attached at half of the beam's length. To assess the robustness of the method with regard to imperfect excitation, the extreme case of a single-point forcing, applied at one third of the beam from the clamping, is used. The model includes the beam, the excitation mechanism consisting of a shaker and a stinger, as well as the controller. Modeling the excitation mechanism is crucial, as it is well-known that the structure-shaker interaction can introduce considerable frequency distortion. As reference, the nonlinear modal characteristics are computed for the autonomous system using the method proposed in [5].

In Figure 1, the modal frequency and damping ratio of the first bending mode are depicted as a function of the displacement at the excitation point. Despite minor deviations, particularly in the identified damping for larger vibrations, the accuracy of the experimental approach is considered good. In ongoing studies, it is investigated whether the accuracy of the experimental technique can be further improved by adjusting the location and increasing the number of excitation points. Moreover, the sensitivity of the results with respect to measurement noise and properties of the controller is investigated.

Conclusions

The proposed nonlinear modal testing method is suitable for extracting the nonlinear modes in accordance with the extended periodic motion concept. The results of the pre-test simulation of a friction-damped beam indicate that the method is robust against imperfect spatial load distribution of the appropriated forcing and structure-shaker interaction. It appears feasible to realize the technique with only a single shaker. As the method relies on standard procedures and equipment for vibration testing (shaker, force and vibration sensors, linear experimental modal analysis, PLL controller), the experimental effort is considered moderate. Future work involves the application to other physical sources of nonlinear damping and the actual experimental realization of the method.

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