A nonsmooth state observer for vibro-impact systems: experimental validation

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Abstract. A state observer that only uses the collision time information has recently been developed for linear time-invariant multibody systems with unilateral constraints. The observer is based on synchronization and makes use of switched geometric unilateral constraints, which generate a unidirectional coupling in a master-slave setup. In presence of uncertainties, such as model inaccuracies or disturbances, an exact reconstruction of the observed state is not possible. As a first step in assessing the robustness of the proposed observer, we present an experimental verification of the observer's performance. Furthermore, we account for dry friction in the observer design.

State observer

Consider a nonsmooth mechanical system consisting of a linear, time-invariant structure subjected to geometric unilateral constraints (leading to impacts) and non-opening frictional contacts. The system under consideration is typically a vibro-impact system with frictional linear guides. Let $\mathbf{q}(t)$ be the generalized coordinates and let $\mathbf{u}(t)$ denote the corresponding generalized velocities. The non-impulsive dynamics is described by

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{u} \\ \mathbf{M}\dot{\mathbf{u}} + \mathbf{C}\mathbf{u} + \mathbf{K}\mathbf{q} &= \mathbf{W}\boldsymbol{\lambda} + \mathbf{W}_T\boldsymbol{\lambda}_T + \mathbf{f}(t) \end{aligned} \tag{1}$$

where the system matrices $\mathbf{K} = \mathbf{K}^{\mathrm{T}} \succ 0$, $\mathbf{M} = \mathbf{M}^{\mathrm{T}} \succ 0$ and $\mathbf{C} \succ 0$ are assumed to be constant and positive definite, and the matrices $\mathbf{W}$ and $\mathbf{W}_T$ are constant. The contact forces $\boldsymbol{\lambda}$ model the non-impulsive constraint forces of the unilateral constraints, whereas the contact forces $\boldsymbol{\lambda}_T$ model the friction forces in non-opening contacts, which obey a Coulomb-type friction law [2]. The matrix $\mathbf{W}$ is composed of the generalized force directions of each unilateral constraint and the matrix $\mathbf{W}_T$ contains the generalized force directions of the non-opening frictional contacts. The system is excited by an external forcing $\mathbf{f}(t)$.

Our goal is to design a state observer that only relies on the collision time information of the observed system. The idea is to consider two identical systems in a master-slave setup and to introduce a unidirectional coupling between the two systems that will lead to synchronization, as illustrated in Figure 2. The coupling consists of a boolean switching function $\chi(t)$, representing the contact state of the master system (the observed plant), i.e. the switching function is 1 for a closed contact and 0 otherwise. The slave system (the state observer) makes use of a switched unilateral constraint, i.e. it closes or opens its contacts depending on $\chi(t)$, such that impacts always occur at the same time instants for both the master and the slave system.

The two systems synchronize if the error dynamics of the state difference between the two systems is asymptotically stable. Let $(\mathbf{q}_m(t), \mathbf{u}_m(t))$ and $(\mathbf{q}_s(t), \mathbf{u}_s(t))$ be two arbitrary solutions of the two systems, where the state of the master system is denoted with a subscript m and the state of the slave system is denoted with a subscript s. By denoting the state difference as $\mathbf{e_q} = \mathbf{q}_m - \mathbf{q}_s$ and $\mathbf{e_u} = \mathbf{u}_m - \mathbf{u}_s$ and using the positive definite Lyapunov function

$$V = \frac{1}{2}\left(\mathbf{e}_\mathbf{u}^{\mathrm{T}}\mathbf{M}\mathbf{e}_\mathbf{u} + \mathbf{e}_\mathbf{q}^{\mathrm{T}}\mathbf{K}\mathbf{e}_\mathbf{q}\right), \tag{2}$$

it can be shown that the time derivative $\dot{V}(\mathbf{e_q}, \mathbf{e_u})$ and jumps at impact times $V^+ - V^- := V(\mathbf{e}_\mathbf{q}^+, \mathbf{e}_\mathbf{u}^+) - V(\mathbf{e}_\mathbf{q}^-, \mathbf{e}_\mathbf{u}^-)$ are both negative semidefinite [1, 3] and that V vanishes as time tends to infinity. Hence the two systems do indeed synchronize.

Experimental results

We implement the synchronization based state observer for a simple 2-DOF mass-spring system. The setup consists of two steel blocks, each mounted on a cart of a linear roller guide, as shown in Figure 1. Each cart is attached to four linear coil tension springs that connect the carts to both ends of the linear guide as well as to each other. All springs are under pretension to prevent buckling during operation. A unilateral constraint is implemented as a limiting stop in form of a massive aluminium block that can be positioned at various locations on the linear guide.

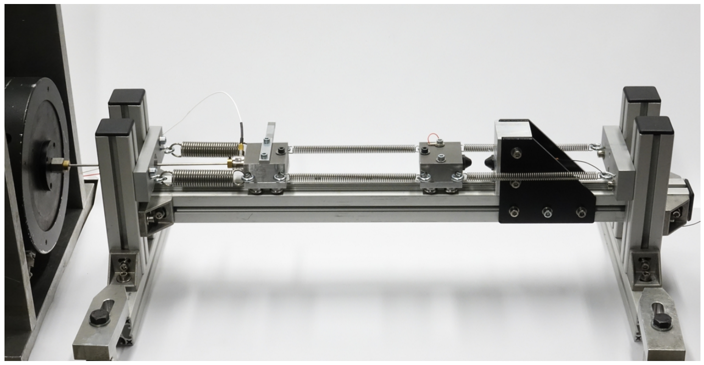

Figure 1: The experimental setup.

The presented state observer shows a good match with the measured state in experiments, compared to a simulation that does not use the collision time information as input (see Figure 2). The state observer output follows the real state under a slowly varying harmonic excitation that changes both amplitude and frequency, while a simulation of the system (using only the measured excitation force as input) alternates between phases of good match and phases of poor state estimation. Letting the observer's position state jump at collision time instants helps to increase the synchronization speed and corrects the position of the colliding body whenever an impact occurs. However, the measured collision time instants contain additional information that has not yet been used: the value of the observer's pre-impact contact distance provides a measure of how closely synchronized the master and the slave system are and could be used in an adaptive parameter and model updating strategy.

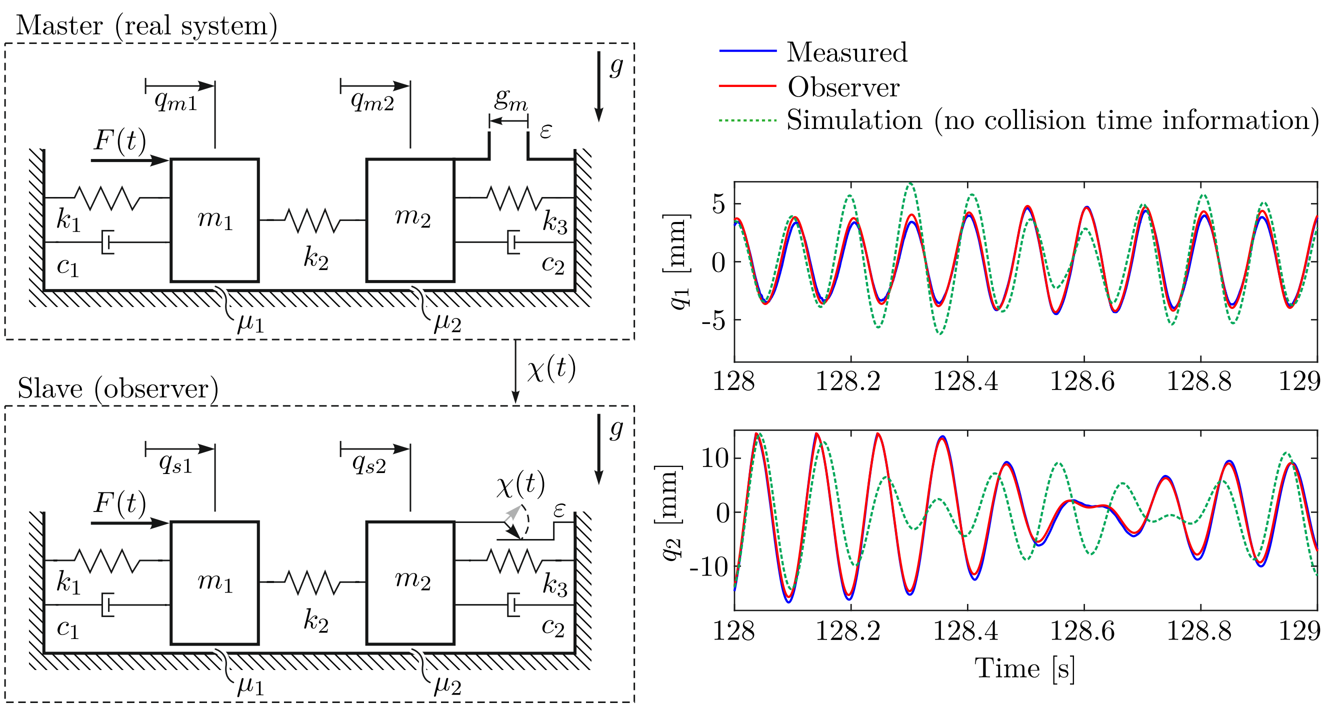

Figure 2: Left: Model of the experimental setup in a master-slave setting. Right: Extract from the time evolution of the measured positions, the observer output and a simulation that only uses the measured excitation force as input.

References

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