Experimental and numerical nonlinear modal analysis of a beam with impact: Part I - Numerical investigation

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Abstract In this contribution we describe a method for numerical nonlinear modal anaylsis of a beam with impact. To model the nonsmooth contact law efficiently a novel approach which combines the shooting and the harmonic balance method is proposed. This approach allows for the solution of contact problems without penalty parameters or restitution coefficients. Due to the conservative nature of this contact law, the method is particularly suitable for the calculation of nonlinear modes of undamped systems. The results are compared to experimentally extracted nonlinear modes obtained in Part II of this study.

Keywords nonlinear modes, Harmonic Balance Method, contact, nonsmooth systems, Finite Element Method

Introduction

Several numerical methods exist to predict the periodic behavior of oscillating structures, e.g., the finite difference method, the shooting method and the harmonic balance method. However, if nonsmooth mechanical systems are considered, then most methods have their difficulties to deal with the set-valued force laws used to describe hard unilateral constraints and Coulomb-type friction.

This contribution proposes a novel approach which combines the shooting and the harmonic balance method in order to handle nonsmooth structural dynamical problems efficiently. The method, which is referred to as the Massless Mixed Shooting-Harmonic Balance Method (MMS-HBM), yields a combined frequency time domain formulation of the equation of motion. For the analysis of finite element models with local unilateral constraints the standard MS-HBM proposed in [1] is augmented by a massless boundary at the contact surface [2].

The numerical method is demonstrated on the example of a laboratory benchmark system consisting of a beam structure with impact at the tip. The calculated nonlinear modes are validated against experimental results obtained with an automated phase resonance approach as described in Part II [3].

Numerical Procedure and Benchmark Structure

In this contribution nonlinear modes are defined as the periodic solutions of autonomous conservative dynamical systems. Hence, for the numerical benchmark system depicted in Figure 1 the definition yields the problem to find the periodic solution of

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) + \mathbf{w}\lambda = 0, \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are obtained by a Finite Element discretization of the beam. Following [2] the mass of the contact node, here the node of the tip of the beam, is neglected. To prevent strong changes of the system characteristics the element length is refined towards the contact node, which makes a complicated redistribution of the mass unnecessary. The massless approach makes it possible to avoid impulsive contact forces. The set-valued signorini contact law can be described using the inequality complementarity condition

$$0 \leq \mathbf{w}^{\mathrm{T}}\mathbf{q} + g_{N,0} \perp \lambda \geq 0. \tag{2}$$

This equation can be rewritten with the proximal point function as an implicit nonlinear equation [4].

We use the Harmonic Balance Method to obtain periodic solutions of Equation (1) which is numerically robust and efficient for large systems with many DOF. However, the harmonic ansatz functions cannot fulfill the non-penetration condition (2) directly. Therefore, we divide the system into a linear and a nonlinear subsystem. The linear subsystem is described in frequency domain and the nonlinear subsystem results in a static equation with inequality condition

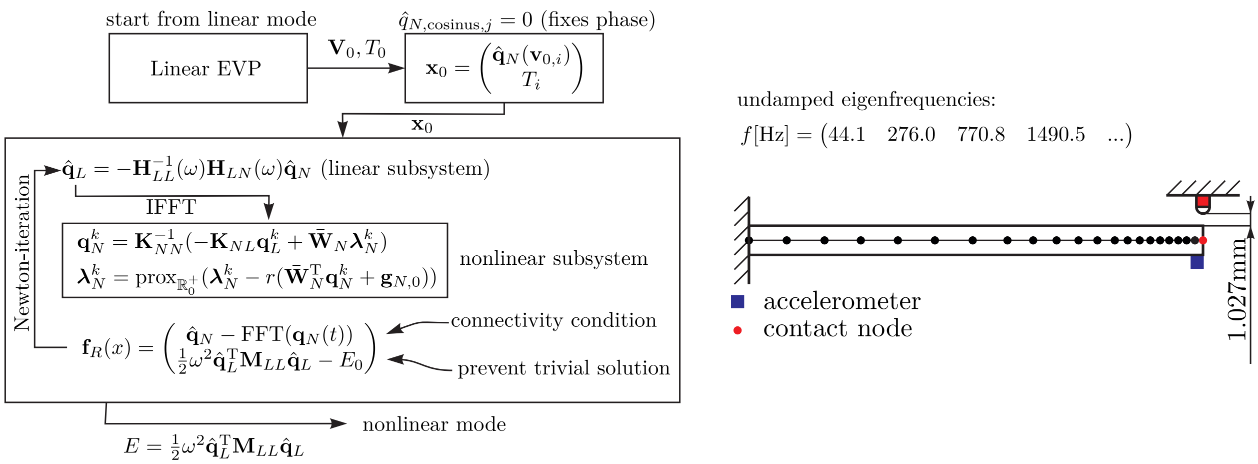

Figure 1: Left: Numerical scheme for the calculation of nonlinear modes. Right: Analyzed finite element model of the beam with impact.

expressed as a proximal point function. The calculation scheme is depicted in Figure 1. The system can be completely described by the vector of unknowns $\mathbf{x}$ which consists of the Fourier coefficients of the nonlinear subsystem and the oscillation period. To find a solution of the problem we use a Newton-type solver which must therefore only iterate in the nonlinear DOF, which are generally only a few compared to the numerous linear DOF. The benchmark system features two nonlinear DOF: the angle and displacement at the contact node, respectively. Additionally, the Jacobian matrix can be calculated semi-analytically, which makes the method also suitable for systems with more nonlinear DOF. The autonomous problem demands a phase anchor which can be considered by setting one Fourier coefficient to zero. Using a prediction-correction pathfollowing technique the branch of a nonlinear modes is calculated starting from the linear eigenfrequency with a low energy level of the analyzed mode.

Numerical Results

The previously explained numerical method is applied to the beam structure with impact and compared against experimental results. Exemplary, the first nonlinear mode is considered. Experimentally, the phase-locked loop (PLL) based nonlinear modal analysis method as described in [3] is applied to the test structure. For the interpretation of the results it is important to note that only the linear parameters of the numerical model, namely the Young's modulus and the density of the material are identified to fit the measured linear eigenfrequencies. Due to the adopted massless approach, no additional contact parameter such as a restitution coefficient or stiffness is employed. Figure 2 shows a comparison of numerical results for two different numerical models with a discretization with 8 and 21 Timoshenko beam elements, i.e., 9 and 22 nodes, respectively. It can be observed that both models yield very similar results. Furthermore, the influence of the number of harmonics considered in the Harmonic Balance calculation is investigated. The results in Figure 2 show that even for comparatively low amplitudes the fundamental harmonic calculation (H1) deviates from the calculations with three (H3) and five (H5) harmonics. An interesting difference between the calculation with three and five harmonics can be observed for higher amplitudes, as the frequency approaches an integer fraction of the second modal frequency (276 Hz), i.e., the system is operated close to a 5:1 internal resonance. The time signal of the displacement and angle of point 3 on the backbone curve already shows a considerable contribution of the fifth harmonic.

An interesting characteristic of the numerical method proposed here is that internal resonances can be analyzed, but similarly to conventional Harmonic Balance approaches, the filtering property is retained. This filtering characteristic, which is not available in pure time domain methods, such as the shooting method, can considerably save computational time. This is particularly interesting when the global behavior of the backbone curve of complex systems with many modal interactions is of interest or in cases where modal interactions do not affect the behavior of the damped system.

Compared to the experimental results measured in Part II a good agreement with the simulations with three and five harmonics can be observed. The experimental curve shows a slightly softer characteristic, i.e., a slightly lower frequency. This can also be observed for the contact force at the second point, which is experimentally slightly lower. An explanation of this difference could be the compliance of the contact element which is neglected in the

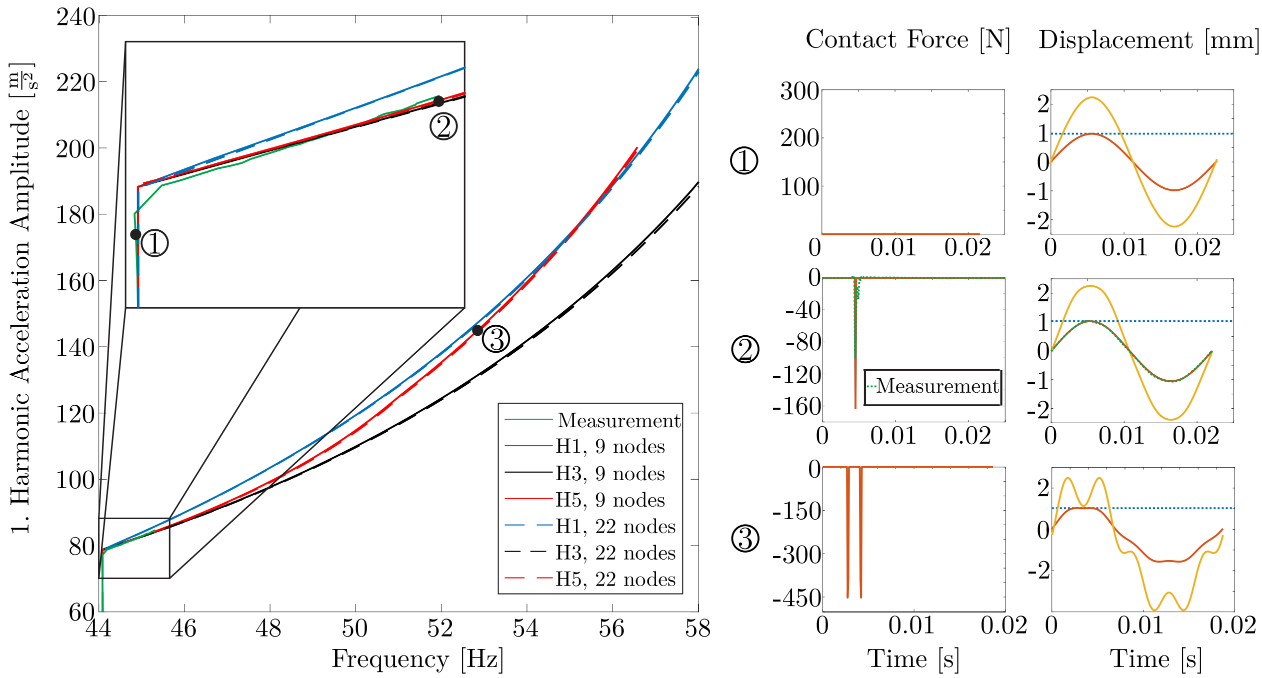

Figure 2: Left: Numerical and experimental backbone curve for the beam tip. Right: Comparison of contact force and displacement for marked points.

numerical model and regarded as rigid. However, the displacement at the tip node shows very good agreement. In the experiment the amplitude range for which the nonlinear mode can be extracted is limited due to the high contact forces and accelerations caused by the hard contact.

Conclusion

This contribution shows that nonlinear modes of systems with unilateral constraints can be calculated with a novel mixed frequency time domain approach which allows to consider nonsmooth contact forces. This approach allows for the solution of contact problems without penalty parameters or restitution coefficients. Due to the conservative nature of the contact law, the method is particularly suitable for the calculation of nonlinear modes, defined as periodic solutions of a conservative autonomous system. The numerical results show a good agreement with the experimental results obtained in Part II of the study. The harmonic ansatz functions which are used to approximate the dynamics of the linear subsystem lead to a numerically efficient method. The filtering characteristic of the harmonic ansatz is essential to calculate nonlinear modes of large finite element systems using a feasible numerical effort.

Further research will be focused on more complex finite element systems and the detailed study of internal resonance phenomena.

References

[1] F. Schreyer and R. I. Leine, A Mixed ShootingHarmonic balance method for unilaterally constrained mechanical systems, in Archive of Mechanical Engineering, Vol. LXIII, No. 2, pp. 297-313, 2016.

[2] F. Schreyer and R. I. Leine, Mixed Shooting-HBM: a periodic solution solver for unilaterally constrained systems, Proc. IMSD, Montral, Canada, 2016.

[3] S. Peter, F. Schreyer, R. I. Leine, Experimental and numerical nonlinear modal analysis of a beam with impact: Part II - Experimental investigation, Proceedings of the 36th IMAC, A Conference and Exposition on Structural Dynamics, 2018, to be published.

[4] R.I. Leine, H. Nijmeijer, Dynamics and Bifurcations of Nonsmooth Mechanical Systems, volume 18. Springer, Berlin, 2004.