University of Stuttgart

Institute for Nonlinear Mechanics

Bachelor's thesis
Master's thesis
Term paper

Numerical Solution of Elastoplasticity via Optimization Methods

Topic Areas:	Nonsmooth Dynamics, Elastoplasticity Numerical optimization
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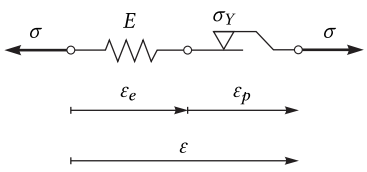
Responsible Professor:	Prof. Remco I. Leine
Prerequisites/Prior Knowledge:	Dynamik mechanischer Systeme (or similar)

At the INM, rockfall protection nets (ring nets) are simulated within the Nonsmooth Dynamics framework. Unilateral contact, Coulomb friction and the elastoplastic behavior of the rings are all written as set-valued force laws, i.e. normal cone inclusions, and discretized by a Moreau–Jean-type scheme [2, 3]. The inclusion problem of each time step is solved by a fixed-point iteration, robust but slow at high accuracy. Convex optimization offers an alternative: common yield criteria are cone representable, so that elastoplasticity becomes a second-order cone complementarity problem (SOCCP) over the Lorentz cone $\mathcal{L}^n$, making standard conic solvers applicable [1]. The hardening element used at the INM likewise leads to an inclusion over a shifted Lorentz cone $\mathcal{L}^2$.

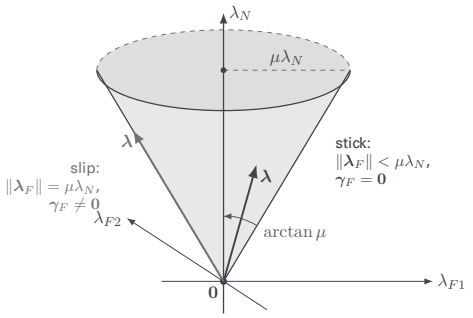
A nonlinear elastoplastic force element with isotropic hardening is to be formulated as a convex second-order cone problem. Contact and friction are included, whereby the non-associated Coulomb law is replaced by a convexified one.



elastplastic force element

The goal is one unified formulation per time step in which contact, friction and plasticity appear as cone conditions of the same type, solvable by several numerical methods. After discretization, the resulting system can be identified as the KKT conditions of a convex quadratic second-order cone program (SOCP), which gives access to a wide range of conic solvers, e.g. interior-point methods. The work is structured as follows.

1. Formulate the elastoplastic force element as a SOC problem.
2. Convexification of friction, and the model errors introduced thereby.
3. Unified formulation: Equation of motion, contact, friction and plasticity combined into one convex SOCP/SOCCP over a Cartesian product of Lorentz cones.
4. Verification. Benchmarks from [2] and a small net example: accuracy, robustness, parameter dependence, computing time.



scaled Lorentz cone $\mathcal{C}^3 = \{(\lambda_N, \boldsymbol{\lambda}_F) \in \mathbb{R} \times \mathbb{R}^2 \,:\, \|\boldsymbol{\lambda}_F\| \leq \mu\lambda_N\}$

[1] L. L. Zhang, J. Y. Li, H. W. Zhang, S. H. Pan: *A second order cone complementarity approach for the numerical solution of elastoplasticity problems*. Computational Mechanics 51(1):1–18, 2013.

[2] L. Eberhardt, J. Breuling, S. R. Eugster, R. I. Leine: *One-dimensional elastoplastic models with isotropic hardening as differential inclusions in primal form*. JTCAM, accepted 2026.

[3] V. Acary, B. Brogliato: *Numerical Methods for Nonsmooth Dynamical Systems*. Springer, 2008.