University of Stuttgart

Institute for Nonlinear Mechanics

Master's thesis

A Total Lagrangian Mixed Kirchhoff–Love Rod Formulation

Topic Areas:	rod theory, numerics, computational mechanics
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Prerequisites/Prior Knowledge:	Dynamik mechanischer Systeme

For the simulation of spatial rods, there exists a multitude of different interpolation strategies required for their corresponding finite element formulation. Especially in the shear-rigid Kirchhoff–Love theory, the objective interpolation of the cross-section-fixed base vectors is not trivial and still part of ongoing research.

In this project, the following interpolation strategy of a two-node element will be explored. The nodal unknowns are given by the Cartesian coordinates of the nodal position together with a non-unit quaternion that parametrizes the nodal cross-section orientation. The length of the non-unit quaternion is further used to model the stretch of the rod. With these nodal unknowns, a C^1-continuous cubic Hermite polynomial can be constructed, which will be used for the centerline interpolation. Moreover, the cross-section-fixed bases along the element should be interpolated using a smallest rotation strategy in combination with a torsional correction. Hence, the required shear-rigidity is exactly enforced by the interpolation strategy.

The first goals of the thesis are to

- derive and implement the corresponding finite element formulation for static analysis
- validate the formulation using a variety of existing static benchmark examples from the literature, including buckling phenomena
- elaborate on a complex showcase example that includes multiple interconnected rods (e.g. a CoreValve™ stent)
- compare the robustness and computational efficiency of the developed formulation to existing rod finite element formulations that weakly enforce the shear-rigidity

Although a Kirchhoff–Love finite element formulation intrinsically shows an absence of shear-locking, the proposed interpolation strategy (as well as existing ones) couples bending, torsion and extension. Hence, the element will be prone to so-called membrane and bending locking. Consequently, further strategies to alleviate these locking phenomena should be investigated. These should include

- an extension of the method to a mixed formulation in which internal forces and moments are treated as independent fields
- testing of different quadrature rules for the spatial integration, e.g., Gauss–Legendre or Gauss–Lobatto