University of Stuttgart

Institute for Nonlinear Mechanics

Bachelor's thesis
Term paper
Master's thesis

Lyapunov-Based Stability Analysis for Efficient Rockfall Simulation

Topic Areas:	nonlinear dyanmics, numerics
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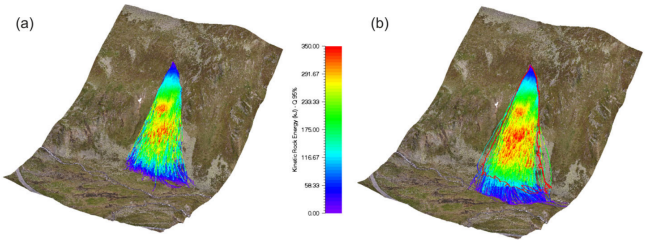
Prerequisites/Prior Knowledge:	Dynamics of Mechanical Systems, Nonlinear Dynamics of Mechanical Systems

The protection of human infrastructure against natural hazards has become a major engineering challenge, especially under the growing impact of climate change. Among these hazards, rockfall poses severe risks to settlements, transportation routes, and critical infrastructure.

Rockfall on Sea to Sky highway (B.C.), taken from [1].

Accurate simulation of rockfall motion requires the correct representation of rigid body rotation and frictional contact with the terrain. The intermediate axis theorem dictates that rotation around the major and minor principal axes is stable, while rotation around the intermediate axis is unstable.

Recently, it is shown in [1] that classically used variants of Moreau's mid-point rule with explicit or linear-implicit evaluation of gyroscopic forces violate these stability properties. The implicit mid-point rule restores the correct behavior, but at the cost of solving a nonlinear system per time step — resulting in simulation times roughly twice as long. This strongly limits large-scale Monte Carlo simulations used to generate reliable hazard maps.



Monte Carlo simulation of 1000 rocks rolling down an actual slope at Chant Sura (Davos, Switzerland) taken from [1]: (a) the explicit scheme, (b) the implicit scheme.

A new integration method has recently been developed at the institute that seems to preserve the physical stability of freely rotating rigid bodies without the computational overhead of the implicit scheme. The goal of this thesis is to prove its stability-preserving properties by constructing a suitable Lyapunov function and performing both analytical and numerical verification.

Specifically, the goals of this thesis are to

- analyze the extended intermediate axis theorem of [1] and its implications for stability.
- find a Lyapunov function proving the stability preservation of the method.
- validate the results numerically.

References

[1] R. I. Leine *et al.*, "Stability of rigid body motion through an extended intermediate axis theorem: Application to rockfall simulation," *Multibody System Dynamics*, 2021.