University of Stuttgart
Institute for Nonlinear Mechanics

Master's thesis

On director-based mixed finite element formulations of geometrically exact shells

Topic Areas: finite elements, structural mechanics

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Prerequisites/Prior Knowledge: dynamics of mechanical systems

Geometrically exact shell models (classical Reissner–Mindlin kinematics) provide a consistent description of thin structures undergoing large rotations and finite strains. In particular, the formulation of Betsch and Sänger [1] provides a C^0 shell element formulation based on a two-field approach with a position ${}_I\boldsymbol{r}_{OC}(\xi_1, \xi_2, t) \in \mathbb{R}^3$ and an additional director field ${}_I\boldsymbol{d}(\xi_1, \xi_2, t) \in \mathbb{R}^3$ describing the shells normal direction. Hence, a material point of the shell can be addressed by

$$ {}_I\boldsymbol{r}_{OP} = {}_I\boldsymbol{r}_{OC} + \xi_3\, {}_I\boldsymbol{d}\,. $$

Although this leads to a very simple finite element formulation, the method is prone to locking in the thin shell limit and leads to differential algebraic equations (DAEs) necessitating highly specialized numerical methods.

Recent developments in nonlinear rod mechanics [2] have shown that mixed Petrov–Galerkin finite element formulations can significantly improve robustness and numerical performance. By introducing additional independent fields in a mixed variational setting, these formulations avoid locking phenomena and lead to stable discretizations for geometrically exact rod models.

The goal of this thesis is to combine these two ideas and extend the mixed Petrov–Galerkin concept to geometrically exact shell formulations. The resulting method is expected to provide improved numerical robustness and flexibility compared to classical displacement-based shell finite elements, particularly in the thin shell limit.

The main objectives of this thesis are to

- formulate a geometrically exact shell model consistent with [1],
- design and implement a Petrov–Galerkin finite element discretization,
- develop a mixed variational formulation inspired by recent rod FE methods [2],
- investigate numerical properties such as stability, convergence, and locking behavior,
- validate the formulation on benchmark problems in nonlinear shell mechanics.

The project is a joint effort between INM & TU Eindhoven and lies at the intersection of nonlinear mechanics, finite element methods, and scientific computing. It involves substantial theoretical and algorithmic development and offers the opportunity to work on a *research-level numerical formulation*.

References

[1] P. Betsch *et al.*, "On the use of geometrically exact shells in a conserving framework for flexible multibody dynamics," *Computer Methods in Applied Mechanics and Engineering*, 2009.

[2] M. Herrmann *et al.*, "A mixed Petrov–Galerkin Cosserat rod finite element formulation," 2025.

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