University of Stuttgart
Institute for Nonlinear Mechanics

Bachelor's thesis
Term paper
Master's thesis

Conic complementarity of contact patches in rigid-body simulation

Topic Areas:	contact mechanics, conic optimization
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Responsible Professor:	Prof. Remco Leine
Prerequisites/Prior Knowledge:	Technische Mechanik, nonsmooth dynamics, numerical optimization

Contacts are everywhere: a robot foot on the floor, a box on a pallet, a stack of bricks. Most rigid-body simulators represent a flat face by its corner points, each obeying the Signorini law $0 \leq \lambda_N \perp v_N \geq 0$. The resulting contact problems are redundant and rank-deficient, the contact forces are non-unique, and solvers suffer from poor conditioning.

A recent paper [1] shows that the Signorini law on a planar contact patch P is equivalent to a single conic complementarity condition on resultant quantities,

$$\mathcal{K}_P \ni (\boldsymbol{m}_T, f_N) \perp (\boldsymbol{\omega}_T, v_N) \in \mathcal{K}_P^* \,,$$

with normal force f_N, tangential moment $\boldsymbol{m}_T$, normal velocity v_N and tilting angular velocity $\boldsymbol{\omega}_T$. The cone $\mathcal{K}_P$ depends only on the patch shape, and the law captures persistent contact, detachment and tipping.

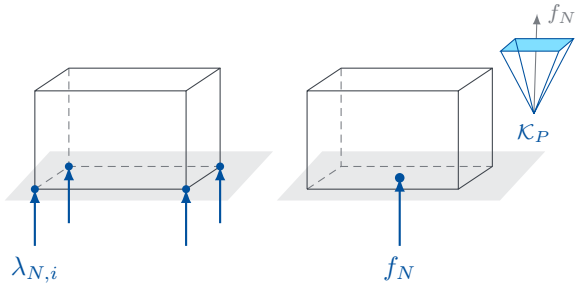

Figure 1: Corner-point contact model (left) versus planar patch contact model (right).

In Moreau–Jean time stepping [2], each patch contributes only three unknowns with a regular Delassus block. Friction, however, is missing: the tangential force and drilling moment depend on the full pressure distribution, while $\mathcal{K}_P$ only retains its first moments.

This thesis project addresses the mathematical formulation of planar patch contact and suitable solution algorithms in the following work packages:

1. Literature review: Signorini law, Moreau–Jean time stepping, Signorini condition [1], patch friction models [3], [4].
2. Formulation of a patch contact element for polygon, segment and point patches: cone $\mathcal{K}_P$, dual cone $\mathcal{K}_P^*$ and patch Jacobian, embedded in Moreau–Jean time stepping.
3. Solution algorithms for the resulting conic complementarity problems, e.g. block projected Gauss–Seidel and interior point methods, and their comparison with the corner-point model.
4. Depending on the course of the thesis project, an extension to frictional contact via limit surfaces [3], pressure reconstruction from $(\boldsymbol{m}_T, f_N)$ or conic formulations [4] should be investigated.

You will work on an open research question at the intersection of nonsmooth mechanics, computational geometry and conic optimization, with direct relevance to robotics.

References

[1] Y. de Mont-Marin *et al.*, *On the conic complementarity of planar contacts*, 2025.

[2] V. Acary *et al.*, *Numerical Methods for Nonsmooth Dynamical Systems*. Springer, 2008.

[3] S. Goyal *et al.*, "Planar sliding with dry friction. Part 1. Limit surface and moment function," *Wear*, 1991.

[4] V. Acary *et al.*, "Coulomb friction with rolling resistance as a cone complementarity problem," *Eur. J. Mech. A/Solids*, vol. 85, 104046, 2021.