University of Stuttgart
Institute for Nonlinear Mechanics

Bachelor's thesis
Term paper
Master's thesis

MPC of a ball on a tilting plate with Moreau–Jean time stepping

Topic Areas:	optimal control, nonsmooth dynamics, mixed-integer optimization
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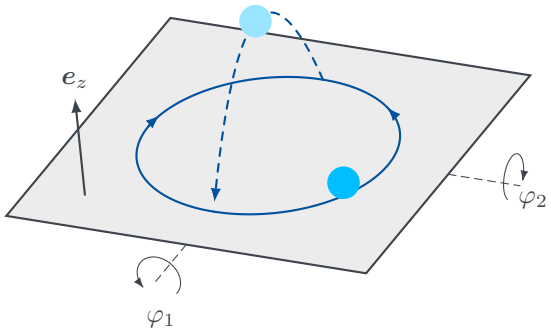
Prerequisites/Prior Knowledge:	numerical optimization, dynamics, Python

Many manipulation tasks require planning when contacts open and close. A simple example is a ball on a tilting plate, inspired by the CyberRunner labyrinth robot [1].

The square plate is tilted about its center and is controlled by actuators on either axis. For the desired trajectory, the ball first follows a half circle, is thrown across the diameter and caught on the other side, and then follows the other half circle. Since the target leaves the plate during the flight, the optimizer must break and re-establish the contact on purpose.



The plant is discretized with the Moreau–Jean time-stepping scheme [2], [3], whose velocity update contains a contact impulse P. The contact law only applies if the contact is closed at the midpoint $\boldsymbol{q}_M$ of the step:

$$\begin{cases} 0 \leq \gamma_{n+1} + e_N \gamma_n \perp P \geq 0\,, & g_N(\boldsymbol{q}_M) \leq 0\,, \\ P = 0\,, & \text{otherwise}\,, \end{cases}$$

with gap g_N, normal gap velocity γ and restitution coefficient e_N.

This if–then–else logic is the core of the thesis project. As a nonlinear program (NLP), it is rewritten with continuous indicators and complementarity constraints. The resulting mathematical program with complementarity constraints (MPCC) is nonsmooth and solved as a sequence of relaxed NLPs. Alternatively, binary lookout and indicator variables with big-M constraints [4] yield a mixed-integer nonlinear program (MINLP), which is combinatorial and needs physically motivated bounds on impulses, velocities and gaps. The modelling choice decides which solvers can be used.

Specifically, the goals of this thesis are

- to model the contact logic of the time-stepping scheme as an MPCC and as an MINLP,
- to implement both in Python with CasADi and solve them with suitable solvers, and
- to compare MPCC and MINLP on the throwing task (solution quality, computation time, robustness, big-M constants) and assess their suitability for model predictive control (MPC).

References

[1] T. Bi *et al.*, "Sample-efficient learning to solve a real-world labyrinth game using data-augmented model-based reinforcement learning," in *Proc. IEEE Int. Conf. Robotics and Automation (ICRA)*, 2024.

[2] J. J. Moreau, "Unilateral contact and dry friction in finite freedom dynamics," in *Nonsmooth Mechanics and Applications*, 1988.

[3] M. Jean, "The non-smooth contact dynamics method," *Computer Methods in Applied Mechanics and Engineering*, 1999.

[4] H. P. Williams, *Model Building in Mathematical Programming*. Wiley, 1999.