University of Stuttgart

Institute for Nonlinear Mechanics

Bachelor's thesis
Term paper
Master's thesis

Backward differentiation formulas for implicit differential equations

Topic Areas:	numerics, algorithms, implementation
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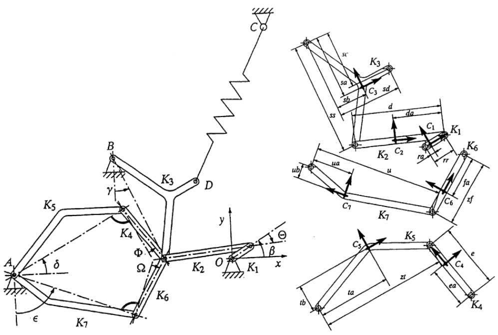
Prerequisites/Prior Knowledge:	interest in numerical algorithms

This thesis explores the numerical time integration of implicit differential-algebraic equations (DAEs) of the form:

$$\boldsymbol{f}(t, \boldsymbol{y}, \dot{\boldsymbol{y}}) = \boldsymbol{0}\,,$$

with given initial conditions. Such systems are fundamental across a wide range of physical modeling applications, including constrained mechanical systems, electrical circuits, incompressible fluids, chemical kinetics with conservation laws, and optimal control problems.



Seven body mechanism benchmark problem.

This thesis concentrates on numerical time integration methods called backward differentiation formulas (BDF). Specifically, the variable order, quasi-constant step-size method proposed in [1] should be investigated due to its simplicity.

To improve the numerical stability of higher-order BDF methods, Klopfenstein [2] introduced numerical differentiation formulas (NDF), which slightly modify BDF methods for enhanced stability. In contrast, the method in [1] attempts to minimize the error constants of these methods without affecting their stability too much. A critical component of these methods is the application of simplified Newton iterations to solve the resulting nonlinear systems efficiently, thereby reducing the number of required Jacobian evaluations and the associated computational overhead.

Furthermore, effective implementation of BDF and NDF methods necessitates the development of sophisticated error estimation techniques and the design of finely tuned adaptive step-size control mechanisms. These components must work in harmony to ensure optimal performance.

The key objectives of this thesis are to

- implement BDF/NDF methods with adaptive step-size control and simplified Newton iterations in Python, similar to the matlab solver proposed in [1].
- validate the implementation using a variety of benchmark problems from the literature that have known analytical solutions
- compare the work-precision of the implemented method with other methods from literature.

The scope of this project may be adapted to fit the specific type of thesis being pursued.

References

[1] L. F. Shampine and M. W. Reichelt, "The MATLAB ODE Suite," *SIAM Journal on Scientific Computing*, 1997.

[2] R. Klopfenstein, "Numerical differentiation formulas for stiff systems of ordinary differential equations," *RCA Review*, 1971.