University of Stuttgart

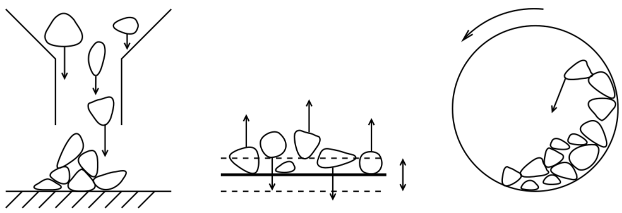
Institute for Nonlinear Mechanics

Bachelor's thesis
Term paper
Master's thesis

Second-order cone programming for frictional contact problems

Topic Areas:	nonsmooth mechanics, optimization algorithms
Advisors:	Jonas Breuling, breuling@inm.uni-stuttgart.de
Responsible Professor:	Prof. Remco Leine
Prerequisites/Prior Knowledge:	Dynamik mechanischer Systeme

Dry Coulomb friction plays a significant role in many mechanical engineering applications. When multiple bodies interact with each other, determining the contact forces in reasonable time becomes essential. This challenge spans various fields, including multibody dynamics, robotic manipulation, deformable solid mechanics, and the study of granular materials.



Examples of contacting bodies.

After discretization of the dynamical systems in space and time, the general model to be considered is a second-order conic complementarity problem (SOCCP) in the De Saxcé form

$$\boldsymbol{f}(\boldsymbol{x},\boldsymbol{\lambda}) = \boldsymbol{0}$$
$$K_\mu^* \ni \hat{\boldsymbol{\gamma}}(\boldsymbol{x}) \perp \boldsymbol{\lambda} \in K_\mu \,,$$

where the contact and friction forces $\boldsymbol{\lambda} = (\lambda_N, \lambda_{T_1}, \lambda_{T_2})^{\mathrm{T}}$ have to be in the Coulomb cone

$$K_\mu = \left\{ \boldsymbol{\lambda} \,\middle|\, \sqrt{\lambda_{T_1}^2 + \lambda_{T_2}^2} \leq \mu\lambda_N \right\}.$$

Moreover, K_μ^* is the dual cone to K_μ and $\boldsymbol{f}$, $\hat{\boldsymbol{\gamma}}$ are nonlinear functions defining the dynamical system.

Currently, mainly first-order numerical methods are available to solve the SOCCP. These methods use the fact that

$$K_\mu^* \ni \hat{\boldsymbol{\gamma}} \perp \boldsymbol{\lambda} \in K_\mu \Leftrightarrow \boldsymbol{\lambda} = \mathrm{prox}_{K_\mu}(\boldsymbol{\lambda} - \rho\hat{\boldsymbol{\gamma}}) \,,$$

where prox_{K_μ} denotes the projection on the set K_μ and ρ is an arbitrary positive parameter. This reformulation allows to perform simple fixed point iterations $\boldsymbol{\lambda}^k \mapsto \boldsymbol{\lambda}^{k+1}$ that can be applied in tandem with a nonlinear solver for $\boldsymbol{f}$. Although these methods are very robust, they suffer from a linear rate of convergence and are therefore too slow to find accurate solutions in a reasonable time.

The main objective of this thesis is to solve the SOCCP using second-order methods to speed up convergence. Therefore, interior-point methods will be investigated as proposed in [1], [2]. This includes the choice of appropriate barrier functions that allow to formulate the SOCCP as equality-constrained optimization problem. In numerical experiments the observed rates of convergence and the obtained speedup with respect to existing first-order methods and semi-smooth Newton methods are analyzed.

References

[1] S. A. Vavasis, K. D. Papoulia, and M. R. Hirmand, "Second-order cone interior-point method for quasistatic and moderate dynamic cohesive fracture," *Computer Methods in Applied Mechanics and Engineering*, 2020.

[2] V. Acary, P. Armand, H. M. Nguyen, and M. Shpakovych, "Second-order cone programming for frictional contact mechanics using interior point algorithm," *Optimization Methods and Software*, 2024.

www.inm.uni-stuttgart.de/en