**University of Stuttgart**

Institute for Nonlinear Mechanics

Master's thesis
Bachelor's thesis
Term paper

# Invariant Cones of Nonsmooth Dynamical Systems

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| Topic Areas: | Nonsmooth Dynamics, Numerical Simulation, Model Order Reduction, Piecewise Linear Dynamics |
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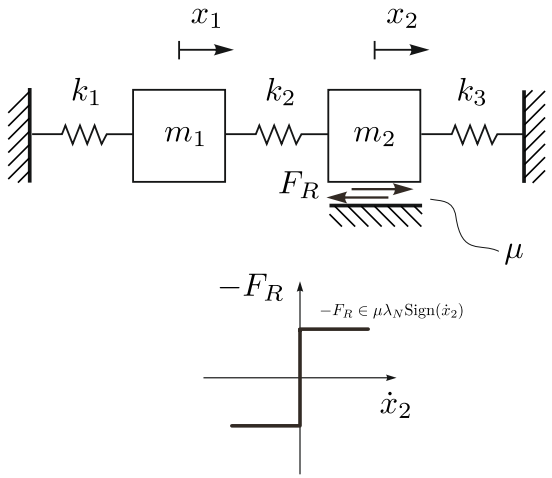
| Responsible Professor: | Prof. Remco I. Leine |
| Prerequisites/Prior Knowledge: | MATLAB |

The concept of nonlinear normal modes (NNMs) is fundamental to understanding the behavior of nonlinear dynamical systems and has been extensively explored for a wide range of system classes. NNMs can be understood as continuations of the classical fundamental solutions of linear systems. They represent invariant sets on which the nonlinear dynamics can be reduced to a lower dimensional system. Therefore, NNMs are very useful for deriving computationally efficient reduced order models. For nonsmooth systems, the task of finding NNMs is more challenging than in the smooth framework due to the lack of differentiability. In a recent work, the concept of invariant cones has been extended to compute the NNMs for a class of nonsmooth systems, which are continuous and piecewise linear (CPL) [1].

In this thesis project, you will investigate discontinuous systems, aiming to establish a similar link between the NNMs in mechanical systems with friction and the invariant cones of discontinuous piecewise linear (DPL) systems. To start this quest, you will learn the theoretical principles of NNMs. Then, you will implement a simple numerical model of a nonsmooth system with Coulomb friction, for which you will compute the NNMs using invariant cones. Once this specific case is well-understood, you will be encouraged to follow an extension path of your choice. Possible ideas include exploring data-driven approaches and applying the method to a higher dimensional FE-model. The overall goals of this project will be adjusted based on the thesis type.




Simple 2-DOF friction oscillator

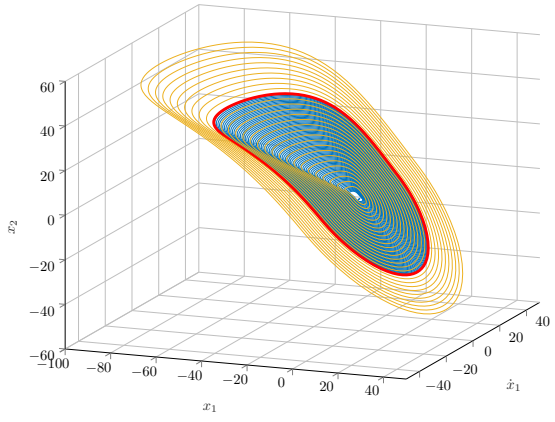



Illustration of invariant cones and NNMs in state space

[1] Karoui, A. Y., & Leine, R. I. (2024). *Extended invariant cones as Nonlinear Normal Modes of inhomogeneous piecewise linear systems. (arXiv preprint arXiv:2407.16096).*