STICK-SLIP WHIRL INTERACTION IN DRILLSTRING DYNAMICS

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Abstract A *Stick-slip Whirl Model* is presented which is a simplification of an oilwell drillstring confined in a borehole with drilling fluid. The disappearance of stick-slip vibration when whirl vibration appears is explained by bifurcation theory. The numerical results are compared with the experimental data from a full-scale drilling rig.

Keywords: drillstring vibrations, discontinuous bifurcations, stick-slip vibrations, whirl, non-smooth systems.

1. Introduction

This paper attempts to explain the complicated behaviour of oilwell drillstring motion when both torsional stick-slip and lateral whirl vibration are involved. It is demonstrated that the observed phenomena in experimental drillstring data could be due to the fluid forces of the drilling mud. A *Stick-slip Whirl Model* is presented which is a simplification of a drillstring confined in a borehole with drilling mud. The model is as simple as possible to expose only the basic phenomena but is discontinuous. Bifurcation diagrams of this discontinuous model for varying rotation speeds reveal discontinuous bifurcations. The disappearance of stick-slip vibration when whirl vibration appears is explained by bifurcation theory. The numerical results are compared with the experimental data from a full-scale drilling rig. A more detailed presentation of the results can be found in [1].

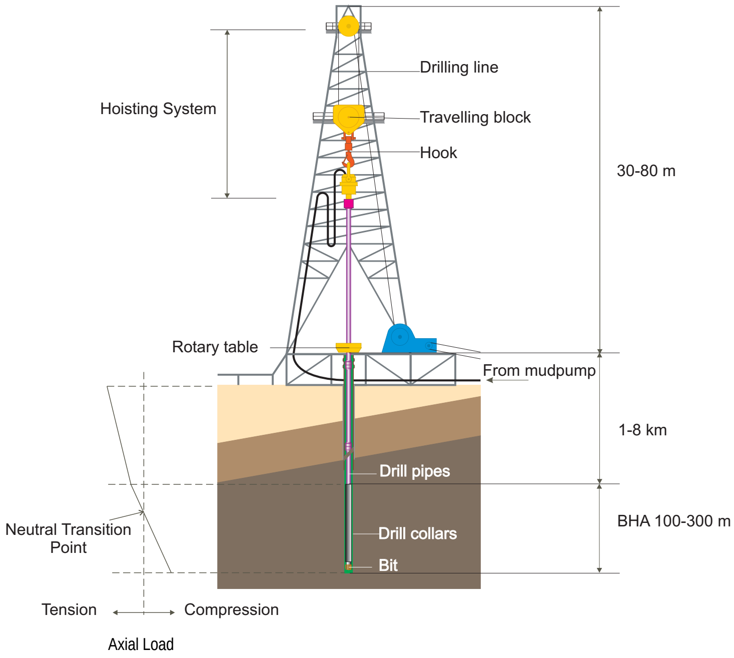


Figure 1. Drilling Rig.

2. Principles of Oilwell Drilling

For the exploration of oil and gas, wells are drilled, which connect the oil/gas reservoir to the surface. The cutting tool to drill those wells is called the drill bit. The bit is turned around with a very slender structure of pipes that are screwed together (see Figure 1). This structure is called the drillstring and can be a few thousand meters long.

To turn the bit, the whole drillstring is rotated from surface with the rotary table (a big flywheel). The lower part of the drillstring is called the Bottom Hole Assembly or BHA and consists of heavier thick-walled pipes, called drillcollars.

On the lower end, the drillstring is resting with the bit on the rock and at the upper end it is pulled upward with a hook at the rig. The slender drillpipe section of the drillstring is therefore constantly in tension while the thick-walled lower part is partly in compression.

Drilling mud, which is a kind of muddy fluid, is pumped through the hollow drillstring by a mudpump. The drilling mud flows through the drillstring, is pumped through nozzles in the bit, and returns to surface in the annulus between drillstring and borehole wall. The function of

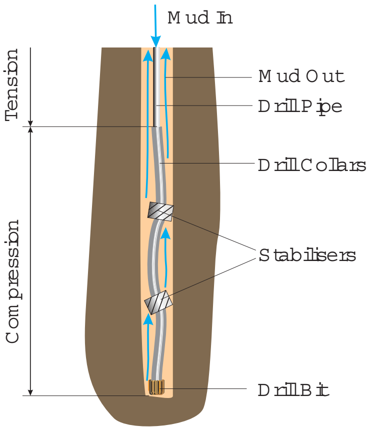


Figure 2. Bottom Hole Assembly.

the mud is to transport the cuttings from the bit to the surface and to lubricate the drilling process.

The tension in the drillpipes avoids buckling of the drillpipe section. The torsional rigidity of the drillpipe section is however very small, due to its length and small wall thickness. The Bottom Hole Assembly is rigid in torsional direction as it is relatively short and thick-walled but experiences lateral deflection due to the compressive force. The drill collars in the BHA are kept in position by a number of stabilizers, which are short sections with nearly the same diameter as the bit. The friction forces on the bit and lower part of the drillstring induce a frictional torque that can cause torsional stick-slip vibrations due to the torsional flexibility of the drillpipes [2]. Similarly, whirl vibrations can exist in the drill collar section [3]. Both types of vibrations are detrimental to the drillstring and lower the rate of penetration.

3. Drilling Measurements

The measurements reported in this paper were recorded at a full-scale drilling rig [1]. The experiments were carried out with a measurement device which was screwed between the bit and the lowest drill collar.

A time history of the rotation speed of the torsionally rigid lower part of the drillstring is depicted in Figure 3. The drillstring is in a torsional stick-slip vibration for the first 30 seconds, during which the Bottom Hole Assembly acts like a torsional pendulum in which one can see the drillpipes as a torsional spring. After 30 seconds, the stick-slip vibration suddenly stops and the rotation speed of the Bottom Hole Assembly

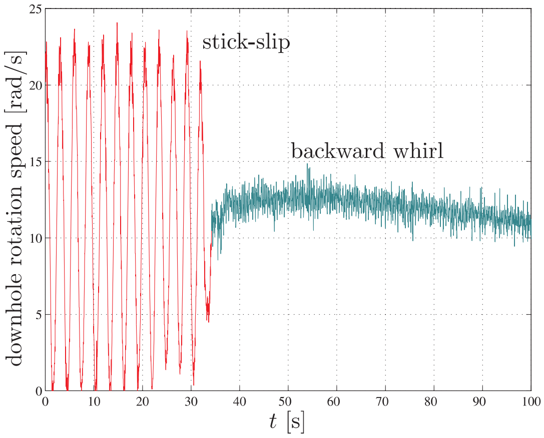


Figure 3. Measured downhole angular velocity versus time.

becomes constant. Whirl vibration is initiated, which is a lateral type of vibration.

The downhole measurement device also measured the bending moment which was exerted on it. The mean bending moment as a function of the downhole rotation speed is depicted in Figure 4. To some extent, the mean bending moment is a measure for the radial deflection of the drillstring. During whirl the mean bending moment is considerably larger than during stick-slip vibration. This indicates that the vibration denoted by 'whirl' in Figure 4 has a much larger lateral deflection and is indeed a whirl-type of vibration.

The frictional torque as a function of the rotational speed is depicted in Figure 5. The frictional torque is corrected for acceleration effects. A kind of friction curve is therefore obtained. During stick-slip vibration the friction curve clearly shows a Stribeck effect which explains the instability of steady rotation. Interestingly, during whirl the friction torque is higher. The additional torque is due to the lateral deflection during whirl. The larger lateral deflection will not only increase the contact between the Bottom Hole Assembly and the borehole wall, but will also increase fluid drag forces on the drillstring that give a kind a viscous friction torque. Therefore, the whirl phase is not only characterized by a higher level of the friction curve, but also by a positive slope of the friction curve due to the viscous friction induced by fluid forces. The fluid forces therefore annihilate the Stribeck effect and constant rotation becomes stable. The whirl vibration therefore excludes the stick-slip motion: the vibration is lateral whirl *or* torsional stick-slip. Stick-slip occurs for low angular velocities and whirl for high angular velocities

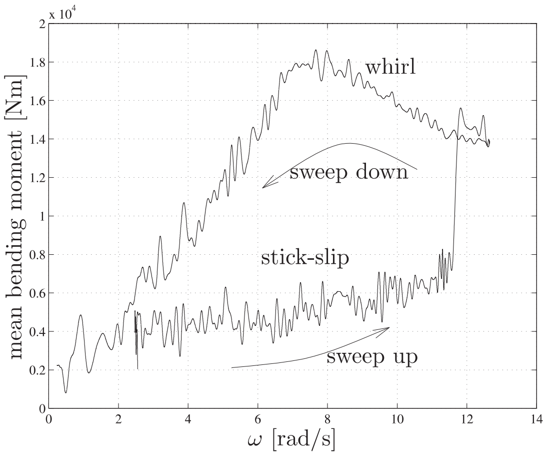


Figure 4. Measured downhole bending moment versus surface angular velocity; sweep-up followed by sweep-down.

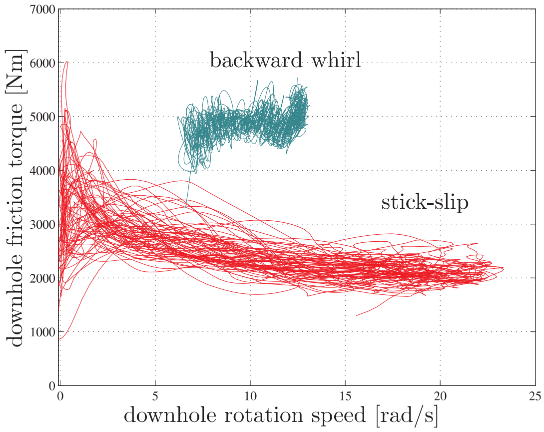


Figure 5. Measured downhole friction curve.

(see Figure 4). There exists a hysteresis phenomenon in between, where the two types of vibration are co-existing stable attractors.

4. Modelling of Stick-slip Whirl Interaction

As a first step, the interaction phenomenon between stick-slip and whirl is modelled as simple as possible (see [1] for a detailed description of the model). The most simple model that can qualitatively describe the observations is a one-DOF model for the torsional vibration and a two-DOF model for the lateral vibration. The total model thus consists of 3 degrees of freedom: the twist and two lateral displacements (Figure 6).

A kind of interaction must exist between the torsional and the lateral model. Mass unbalance can explain the occurrence of whirl, but the whirl would then only be present for angular velocities near a resonance frequency. Instead, we observe that whirl takes place for angular velocities above a critical value. It is therefore assumed that fluid forces form the interaction mechanism between torsional motion and lateral motion.

The fluid forces are modelled as simple as possible by equations that are also used in the theory for full film bearings [4]

$$\begin{aligned} F_{fr} &= -m_f(\ddot{r} - \dot{\alpha}^2 r - \tfrac{\dot{\phi}^2}{4} r + \dot{\phi}\dot{\alpha} r) - (D + \psi_2(r))\dot{r} - \psi_1(r) r, \\ F_{f\alpha} &= -m_f(\ddot{\alpha} r + 2\dot{r}\dot{\alpha} - \dot{\phi}\dot{r}) - (\dot{\alpha} - \tfrac{\dot{\phi}}{2})(D + \psi_2(r)) r. \end{aligned} \tag{1}$$

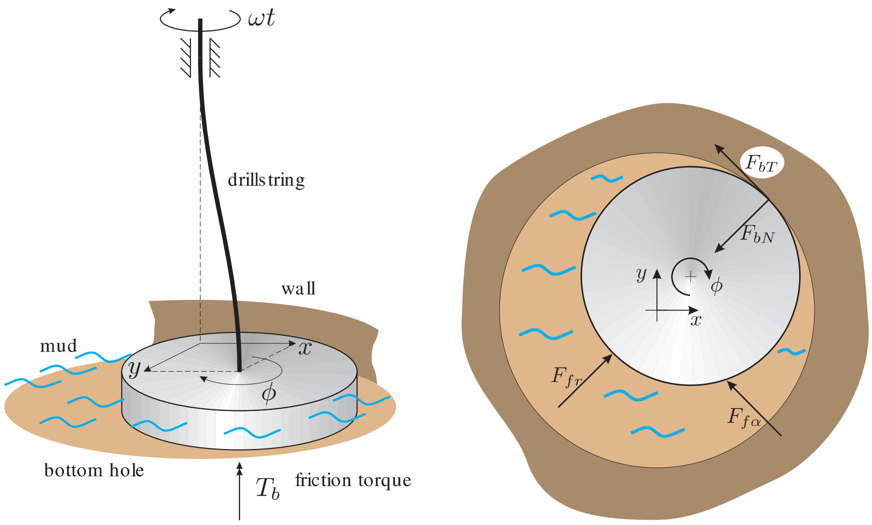
The fluid motion results in a lift force F_{fr} in radial direction on the rotor and a drag force $F_{f\alpha}$ in the tangential direction (Figure 6b). The constant m_f is the added mass of the fluid and D is a damping parameter. The nonlinear functions $\psi_1(r)$ and $\psi_2(r)$ constitute the higher-order terms. The whirl speed is denoted by $\dot{\alpha}$ and the rotation speed of the disk by $\dot{\phi}$. The terms $m_f\dot{\alpha}^2 r$ and $m_f\frac{\dot{\phi}^2}{4} r$ are forces which push the drillstring to the wall. These terms are very important as they are the cause of an instability effect as will be explained in the sequel.

With numerical continuation methods, we are able to study the nonlinear dynamics of the Stick-Slip Whirl Model. First, only the Whirl Model is investigated.

4.1 Whirl Model

The Whirl Model contains only the 2 lateral degrees of freedom of the Stick-Slip Whirl Model. The bifurcation diagram of the Whirl Model is depicted in Figure 7, with the rotational speed as bifurcation parameter and the lateral deflection on the vertical axis.

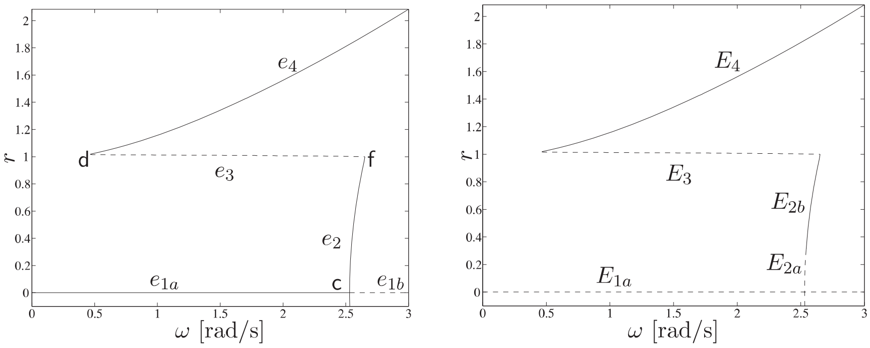
For small rotational speeds the drillstring is in a trivial stable equilibrium position with no deflection. If the rotational speed is increased then



(a) Model.

(b) Fluid forces.

Figure 6. Stick-slip Whirl Model.



(a) Whirl Model.

(b) Stick-slip Whirl Model.

Figure 7. Equilibrium branches.

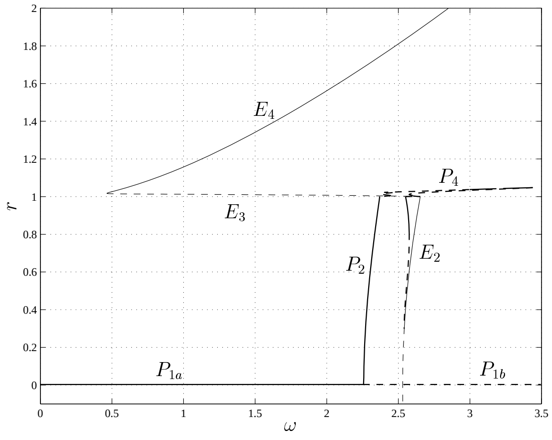


Figure 8. Stick-slip Whirl Model, periodic branches (bold).

the fluid forces increase and a pitchfork bifurcation occurs at point c. The trivial equilibrium branch is unstable for high rotational speeds because the destabilizing effect of the fluid forces is larger than the restoring elastic forces.

For rotational speeds just after point c the drillstring starts to whirl in forward direction. This whirl motion is a periodic solution in a frame fixed to the world, but is an equilibrium in a co-rotating frame of reference. All branches in the bifurcation diagram are therefore equilibria because a co-rotating frame was used for the Whirl Model.

The amplitude of the whirl motion rises until the borehole wall is hit. Bifurcation points d and f are discontinuous saddle-node bifurcations of the equilibrium branch $e_2 - e_3 - e_4$. Co-existing is a branch with stable backward whirling solutions. During backward whirl the drillstring is rolling over the borehole wall. An unstable branch connects the branch of stable forward whirl solutions with the branch of stable backward whirl solutions. Note that for very small rotational speeds the Whirl Model is in its trivial equilibrium position, whereas it is in backward whirl for very high rotational speeds. A kind of hysteresis phenomenon exists in between.

4.2 Stick-slip Whirl Model; bifurcation diagrams

We now study the same equilibrium branches, but for the full Stick-Slip Whirl Model which also includes the torsional degree of freedom. As the lateral and the torsional model are uncoupled to some extent, all equilibrium branches of the Whirl Model are also equilibria of the Stick-

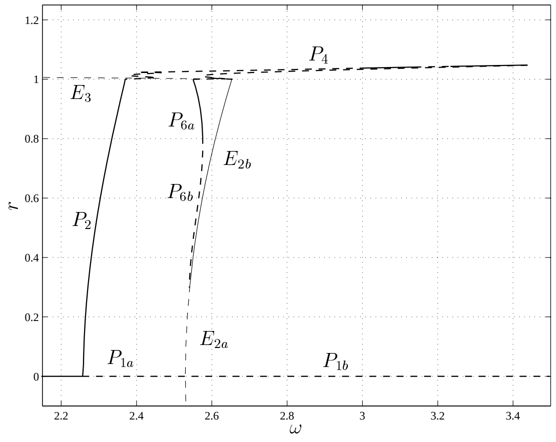


Figure 9. Stick-slip Whirl Model, zoom of Figure 8.

slip Whirl Model (something which is not true in general). The stability of the equilibrium branches, however, changes due to the torsional degree of freedom.

The disk in the Stick-Slip Whirl Model is a very simple model of the Bottom Hole Assembly and a dry friction torque T_b is assumed to act on it, which models the bit-rock interaction (as well as the stabilizer wall contact). The Stribeck effect in the friction curve destabilizes the trivial equilibrium (curve E_1) of the pure Whirl Model. Also a part of the forward whirl branch is unstable: branch E_2 contains a Hopf bifurcation.

The full bifurcation diagram of the Stick-Slip Whirl Model, showing the equilibrium branches in a co-rotating frame as well as the periodic branches, is depicted in Figure 8.

For small rotational speeds the full Stick-Slip Whirl Model undergoes a stable torsional stick-slip oscillation. If the rotational speed is increased, then the fluid forces destabilize the undeflected trivial position and the drillstring will deflect in lateral direction. When the drillstring is laterally deflected the drillstring will act like a stirring spoon in a cup of tea. The torsional stick-slip vibration will therefore be damped due to the viscous fluid friction. The branch E_4 of stable backward whirling solution is unaffected by the torsional model.

For small rotational speeds the Stick-Slip Whirl Model is in torsional stick-slip oscillation, whereas it is in lateral backward whirl motion for high rotation speeds. A very complicated hysteresis region exists in between, where both types of motion can be present as well as many other mixed and period doubled types of oscillation. This region with very complicated dynamic behavior is, however, localized.

A zoom of the hysteresis region is depicted in Figure 9. Periodic branches are created at points where the equilibrium branches lose stability. From the Hopf bifurcation at the equilibrium branch E_2 emanates a periodic branch P_6. Branches P_2 to P_6 connect the solutions found from the Whirl Model with the solutions found from the pure Stick-Slip Model (see [1] for details). The bifurcation structure is apparently very complicated. Branch P_4 undergoes a number of period doubling bifurcations. The period doubled solutions have not been calculated. A number of bifurcation points are smooth, while other bifurcation points are non-smooth. These non-smooth bifurcation points behave sometimes like classical smooth bifurcation points, but can also show a behavior which is non-standard [5].

5. Conclusions

We concluded from the measurements on a real drillstring that torsional stick-slip and lateral whirl can be co-existing attractors in oilwell drilling. The presented Stick-slip Whirl Model is a highly simplified model of reality that describes the observed phenomena in its most simple form. A bifurcation analysis of the Stick-slip Whirl Model revealed non-standard bifurcations, which are due to the non-smoothness of the system.

Acknowledgments

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