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Combined Frequency-Time Reduction Methods for Calculating Periodic Solutions of Unilaterally Constrained Systems

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Several numerical methods exist to predict the periodic behavior of coupled oscillating structures, e.g., the shooting method and the harmonic balance method. The harmonic balance method (HBM) is very popular as it is well suited for large systems with many DOF. However, if nonsmooth mechanical systems are considered, then most methods have their difficulties to deal with the set-valued force laws, used to describe hard unilateral constraints and Coulomb-type friction. In this contribution we describe a novel approach which combines the shooting and the harmonic balance method, which is referred to as the Mixed-Shooting Harmonic Balance Method (MS-HBM) [1]. To find the periodic response of

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{D}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) + \mathbf{W}\boldsymbol{\lambda} = \mathbf{f}_e(t),$$

we divide the **M**-**D**-**K**-system into linear and a nonlinear subsystems. As benchmark system (Figure 1) we use a chain of masses which are connected with linear force elements. Only the n-th mass is subjected to nonlinear forces induced by a unilateral constraint. The nonlinear part is marked with the red cycle and consists of all DOF which are subjected to nonlinear forces, whereas the other masses are part of the linear subsystem. From Hamilton's principle of varying action we derive a combined frequency-time reduction method. Therefore, global ansatz functions in time are chosen for the linear subsystem. Hence, its periodic response can be approximated using a small harmonic basis. Since this global ansatz functions cannot fulfill the set-valued force laws, the DOF of the nonlinear subsystem are discretized with local linear ansatz functions in time. Finding the periodic solutions of the discretized equations boils down to find the zeros of a residuum function $\mathbf{f}_R$, which consists of a periodicity and a compatibility condition. The first ensures a periodic oscillation of the nonlinear subsystem and the second guarantees that both subsystems oscillate consistent to each other. Since $\mathbf{f}_R$ does not depend on the size of the linear subsystem the MS-HBM can significantly reduce the numerical effort in comparison to the shooting method.

As depicted in Figure 2 a) the impulsive dynamics of the n-DOF-oscillator can be well approximated, considering only a few harmonics of the linear subsystem. In comparison, the HBM with penalty approach cannot model the impulsive character of the unilateral constraint and the shooting method becomes unfeasible for large linear subsystems.

If finite element systems are considered, the nonlinear subsystem can consist of many DOF. Since the MS-HBM requires a numerically obtained Jacobian matrix the numerical effort increases dramatically with the size of the nonlinear subsystem. Additionally, a fine discretization of the contact area results into small masses which makes an impact law not appropriate. Therefore, the MS-HBM is augmented using a massless boundary approach [2], which allows to solve the contact problem without the need of penalty

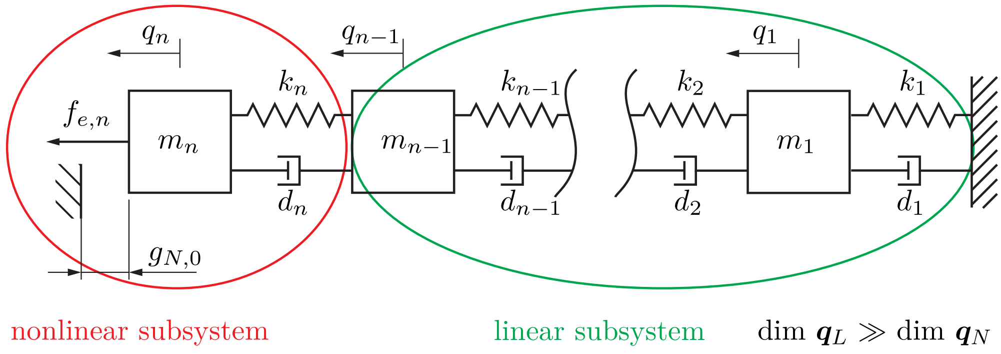

Figure 1: n-DOF-oscillator with unilateral constraint.

parameters or restitution coefficients and the Jacobian matrix can be calculated semi-analytically. In Figure 2 b) the frequency response function of the contact node of a finite element discretized bar is depicted. The bar is fixed on one end and excited with an harmonic force. We use Moreau's time-stepping technique with an inelastic impact law ($e = 0$) as reference. The results show good agreement between the massless MS-HBM with 10 considered harmonics and the time integration.

Due to the conservative property of the contact law, the method is particularly suitable to analyze conservative systems and can therefore be an interesting method for the calculation of nonlinear normal modes.

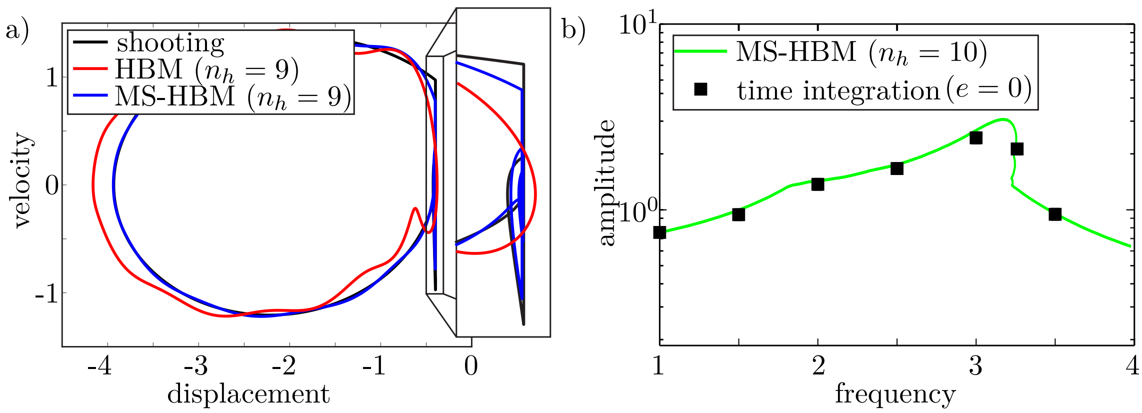

Figure 2: a) Phase portrait of nonlinear mass of 50-DOF oscillator. b) Frequency response function of an unilaterally constraint finite element bar.

References

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[2] Schreyer, F.; Leine R. I.: Mixed Shooting-HBM: a periodic solution solver for unilaterally constrained systems. Proc. IMSD, Montreal, Canada, 2016.